Sustainable Water Resources Research Center

Modeling of 2D Flood Flow Analysis by Finite Element Method

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Kun-Yeun Han, Sang-Ho Kim, Seung-Yong Choi

Kyungpook National Univ., Korea

Outline

1. Introduction
2. Development of RAM2 Model
3. Model Calibration and Verification
4. Applications of RAM2 to Natural Rivers
5. Conclusions

Introductions

- 2D hydrodynamic model can provide good estimates of complex features in the flow around islands and obstructions, flow at confluence and flow in braided channel.

- The challenging problem facing two-dimensional FE model is the treatment of wet and dry areas. This situation is encountered in most practical river such as flood propagation, dam break analysis, tidal processes and so on.

Objectives of the Study

- To develop an accurate and robust two-dimensional finite element method with wet and dry simulation in complex natural rivers.

Scope of the Study

Development of Dry/Wet Algorithm

- Development of Deforming Grid Method
- Development of Transition Element Method
- Development of Hybrid Method

Application of Dry/Wet Algorithm

- Laboratory U-shaped Channels
- 2D Dambreak-wave, Several cases with dry state

Application of RAM2 Model to natural channel

- Application of RAM2 Model for Milyang-river, Nakdong-river, Kum-river

Wateshed Analysis in Hydrology/Hydraulic Aspect

Analysis Model of Hydrology in Watershed

Numerical River Analysis Model adopted Wet/Dry Treatment

Deforming Grid Method

Transition Element Method

Hybrid Method

Numerical Analysis Results

Analysis of River Convection-Diffusion

Analysis of River Bed Variation

Connected with GUI

α- test and β- test

River Model Development

Estimation of Water Budget and Mass Budget

Practical Use of Sustainable Water Resources Research Establishment

Governing Equation (Matrix Form)

$$\frac{\partial U}{\partial t}+\frac{\partial F}{\partial x}+\frac{\partial G}{\partial y}+D=0$$

$$U=(h,p,q)^{T}$$

$$F=\begin{pmatrix} p \\ \dfrac{p^{2}}{h}+\dfrac{gh^{2}}{2} \\ \dfrac{pq}{h} \end{pmatrix}=M_{x}U=\begin{bmatrix} 0 & 1 & 0 \\ \dfrac{gh}{2} & u & 0 \\ 0 & 0 & u \end{bmatrix}U \qquad G=\begin{pmatrix} q \\ \dfrac{pq}{h} \\ \dfrac{q^{2}}{h}+\dfrac{gh^{2}}{2} \end{pmatrix}=M_{y}U=\begin{bmatrix} 0 & 0 & 1 \\ 0 & v & 0 \\ \dfrac{gh}{2} & 0 & v \end{bmatrix}U$$

$$D=\begin{pmatrix} i \\ gh\dfrac{\partial z_{0}}{\partial x}+gn^{2}\dfrac{p(p^{2}+q^{2})^{1/2}}{h^{7/3}}-\dfrac{\partial}{\partial x}\left(\varepsilon_{xx}\dfrac{\partial p}{\partial x}\right)-\dfrac{\partial}{\partial y}\left(\varepsilon_{xy}\dfrac{\partial p}{\partial y}\right)-\dfrac{p}{h}\left(\dfrac{i}{2}\right) \\ gh\dfrac{\partial z_{0}}{\partial y}+gn^{2}\dfrac{q(p^{2}+q^{2})^{1/2}}{h^{7/3}}-\dfrac{\partial}{\partial x}\left(\varepsilon_{yx}\dfrac{\partial q}{\partial x}\right)-\dfrac{\partial}{\partial y}\left(\varepsilon_{yy}\dfrac{\partial q}{\partial y}\right)-\dfrac{q}{h}\left(\dfrac{i}{2}\right) \end{pmatrix}$$

Weighting Functions

$$\int_{\Omega} B_i^T \left(\frac{\partial \hat{U}}{\partial \tau} - \frac{\partial x}{\partial \tau}\frac{\partial \hat{U}}{\partial x} - \frac{\partial y}{\partial \tau}\frac{\partial \hat{U}}{\partial y} + A\frac{\partial \hat{U}}{\partial x} + B\frac{\partial \hat{U}}{\partial y} + D \right) d\Omega = 0$$

Weighting Functions

Bubnov-Galerkin Scheme

$$[B_i^{'}] = [B_i]$$

Petrov-Galerkin Scheme

$$[B_i^{'}] = [B_i] + \varepsilon_x \left[[A]^T \frac{dB_i}{dx} \right] + \varepsilon_y \left[[B]^T \frac{dB_i}{dx} \right]$$

$$\varepsilon_x = \frac{\Delta x}{|U + c|\sqrt{15}}$$

$$\varepsilon_y = \frac{\Delta y}{|V + c|\sqrt{15}}$$

$$\varepsilon = \frac{2}{\sqrt{15}} \approx 0.50$$

SU/PG Scheme

$$[B_i^{'}] = [B_i] + \alpha \frac{\Delta x}{2} \left[\left[\frac{[A]}{|[A]|} \right]^T \frac{dB_i}{dx} \right] + \alpha \frac{\Delta y}{2} \left[\left[\frac{[B]}{|[B]|} \right]^T \frac{dB_i}{dx} \right]$$

$$[Wx] = [Mx] \left[\frac{\lambda_i}{|\lambda_i|} \right] [Mx]^{-1}$$

$$[Wy] = [My] \left[\frac{\lambda_i}{|\lambda_i|} \right] [My]^{-1}$$

Upwinding Matrix

Upwinding Matrix of PG Scheme(2D)

$$W_x = \begin{bmatrix} 0 & \frac{1}{|U+c|} & 0 \\ \frac{c^2-U^2}{|U+c|} & \frac{2U}{|U+c|} & 0 \\ \frac{-UV}{|U+c|} & \frac{V}{|U+c|} & \frac{U}{|U+c|} \end{bmatrix}$$

$$W_y = \begin{bmatrix} 0 & 0 & \frac{1}{|V+c|} \\ \frac{-UV}{|V+c|} & \frac{V}{|V+c|} & \frac{U}{|V+c|} \\ \frac{c^2-V^2}{|V+c|} & 0 & \frac{2V}{|V+c|} \end{bmatrix}$$

Upwinding Matrix of SU/PG(2D)

$$[W_x] = -\frac{1}{2c}\begin{bmatrix} 1 & 0 & 1 \\ U-c & 0 & U+c \\ V & 1 & V \end{bmatrix}\begin{bmatrix} \frac{U-c}{|U-c|} & 0 & 0 \\ 0 & \frac{1}{2}\left(\frac{U+c}{|U+c|}+\frac{U-c}{|U-c|}\right) & 0 \\ 0 & 0 & \frac{U+c}{|U+c|} \end{bmatrix}\begin{bmatrix} -(U+c) & 1 & 0 \\ 2cV & 0 & -2c \\ U-c & -1 & 0 \end{bmatrix}$$

$$[W_y] = \frac{1}{2c}\begin{bmatrix} 1 & 0 & 1 \\ U & 1 & U \\ V-c & 1 & V+c \end{bmatrix}\begin{bmatrix} \frac{V-c}{|V-c|} & 0 & 0 \\ 0 & \frac{1}{2}\left(\frac{V-c}{|V-c|}+\frac{V+c}{|V+c|}\right) & 0 \\ 0 & 0 & \frac{V+c}{|V+c|} \end{bmatrix}\begin{bmatrix} (V+c) & 0 & -1 \\ -2cU & 2c & 0 \\ -(V-c) & 0 & 1 \end{bmatrix}$$

Implementation of SU/PG Method

Implementation of SU/PG for Governing Equation

$$\begin{bmatrix} B_i & 0 & 0 \\ 0 & B_i & 0 \\ 0 & 0 & B_i \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} + \alpha \Delta x [W_x] \frac{\partial}{\partial x} \begin{bmatrix} B_i & 0 & 0 \\ 0 & B_i & 0 \\ 0 & 0 & B_i \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} + \alpha \Delta y [W_y] \frac{\partial}{\partial y} \begin{bmatrix} B_i & 0 & 0 \\ 0 & B_i & 0 \\ 0 & 0 & B_i \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} = \{0\}$$

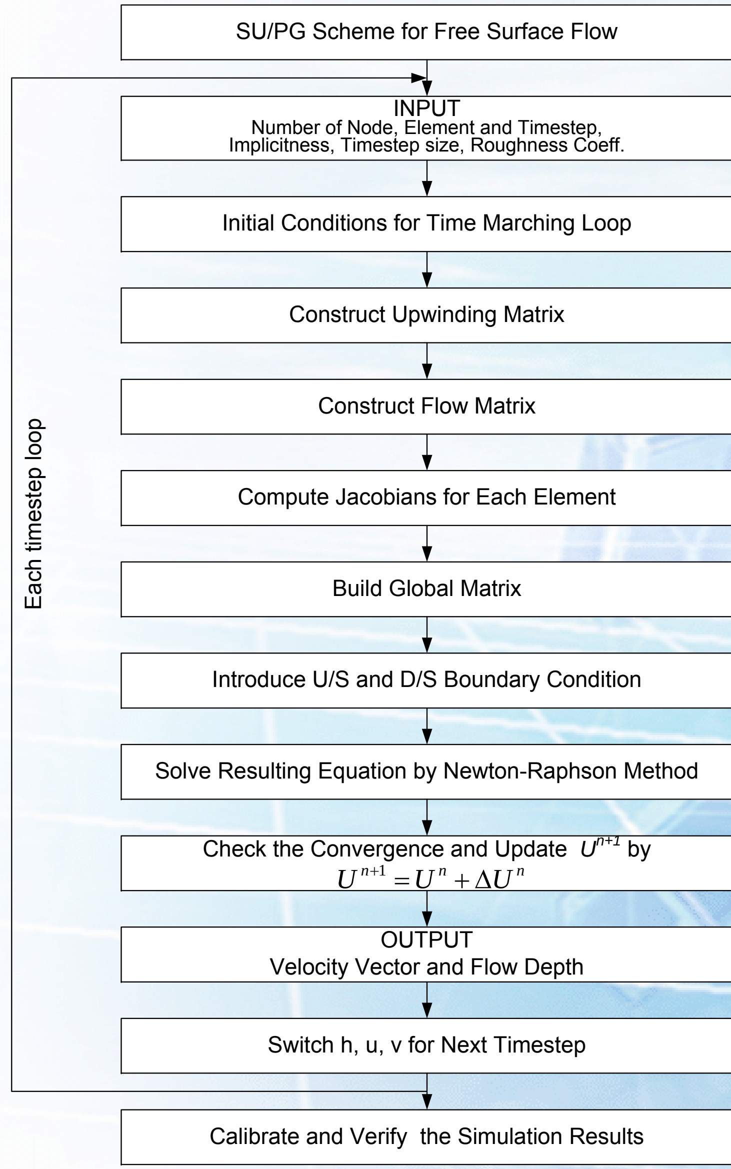
Implementation of SU/PG for Governing Equation (Matrix Form)

$$S_{ij} \frac{dU_i}{dt} + (K_{ij} + \overline{BK_{ij}}\,\overline{CBC}) U_j + \overline{BK_{ij}}\,\overline{BC}\,\hat{U}_j = 0$$

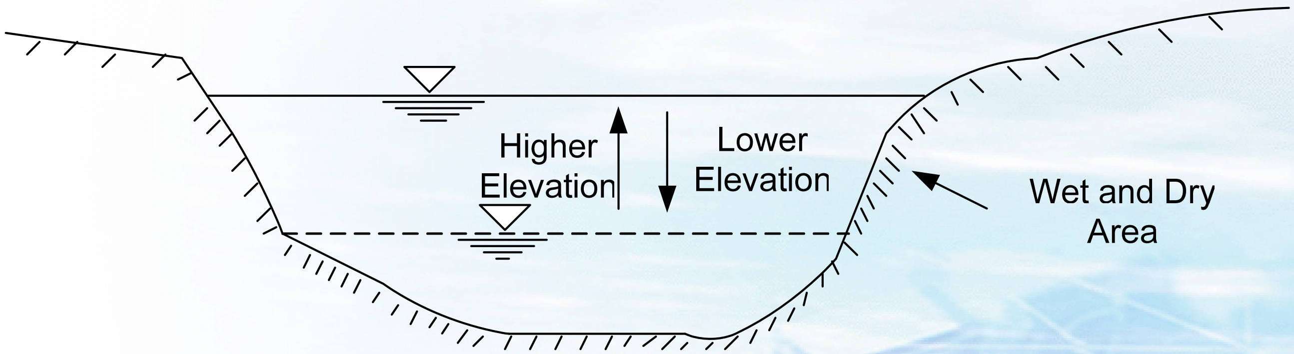
$$S_{ij} = \int_\Omega (B_i + w \Delta x W_x \frac{\partial B_i}{\partial x} + w \Delta y \frac{\partial B_i}{\partial y}) B_j d\Omega$$

$$K_{ij} = \int_\Omega -\frac{\partial B_i}{\partial x} M_x B_i - \frac{\partial B_i}{\partial y} B_j + B_i D B_j + w(\Delta x W_x \frac{\partial B_i}{\partial x} + \Delta y W_y \frac{\partial B_i}{\partial y})(A \frac{\partial B_i}{\partial x} + B \frac{\partial B_i}{\partial y} + N B_j) d\Omega$$

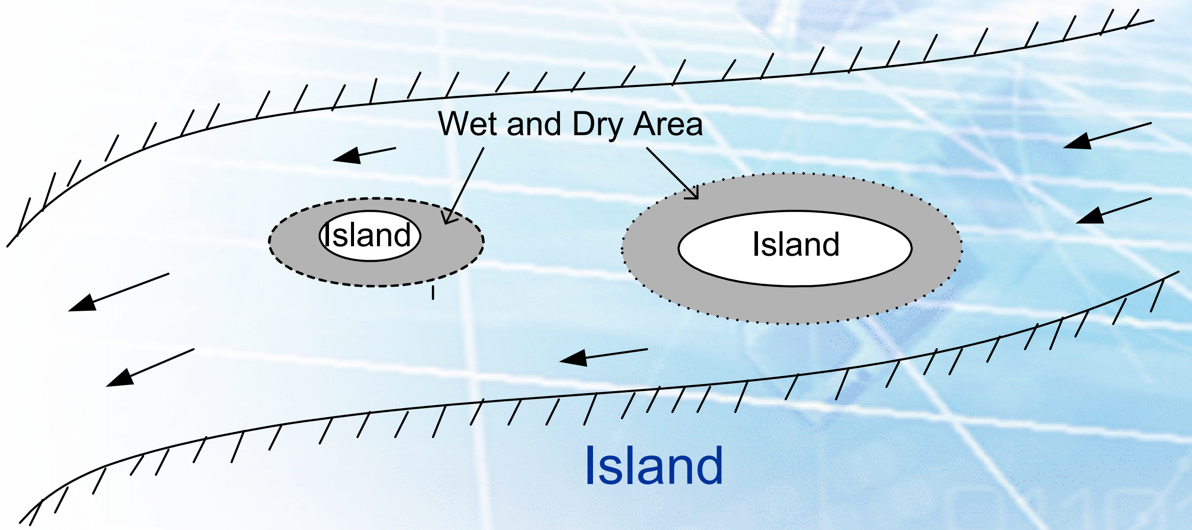
Computational Flowchart



Dry/Wet Problems in Natural River



Floodplain

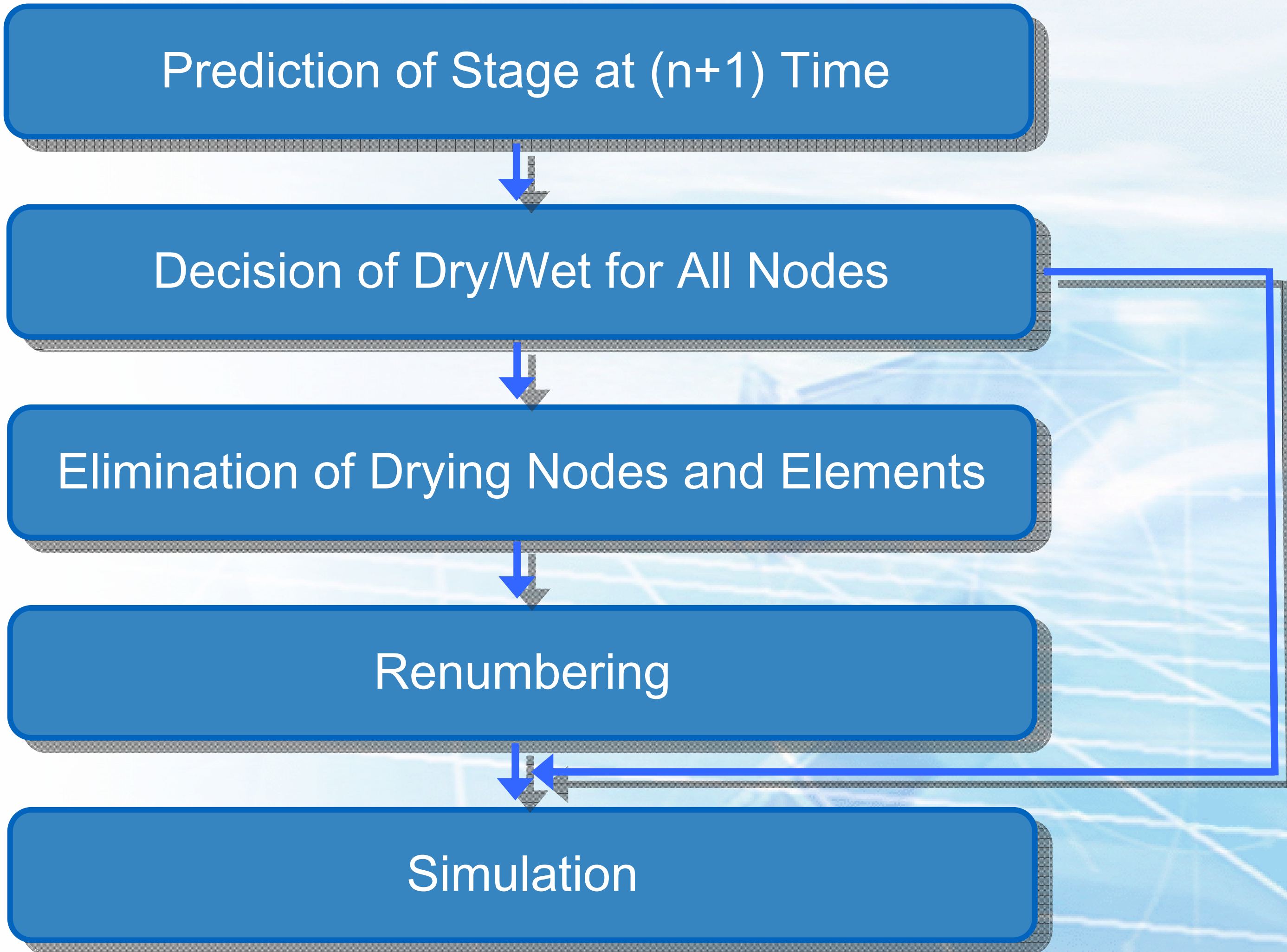


Island

Deforming Grid Method

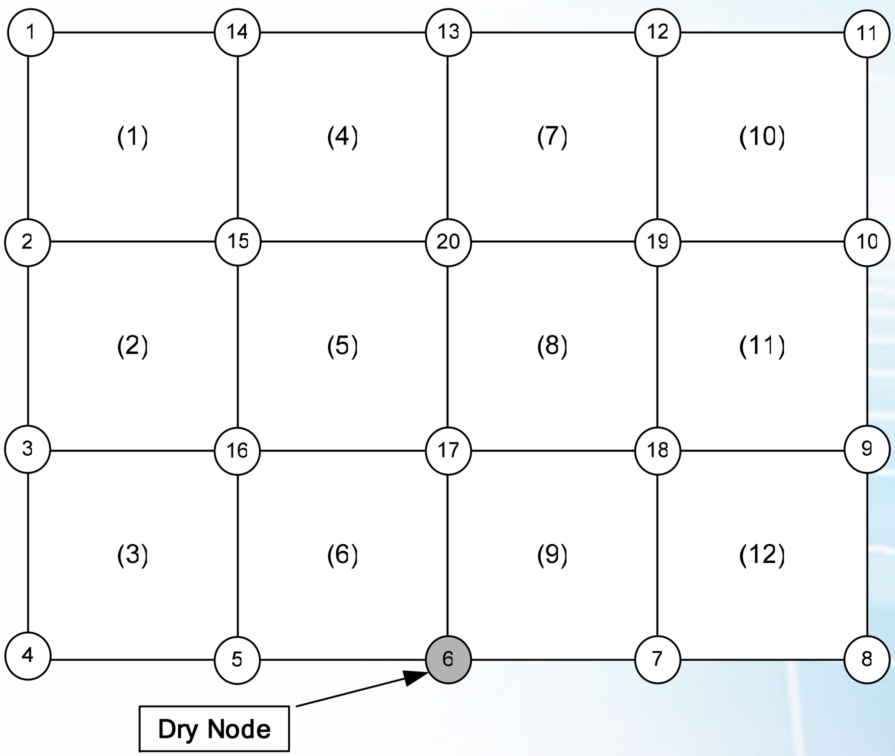
- Prediction of stage at (n+1) time by considering the rising rate of water level
- Decision of Dry/Wet for all nodes
- Elimination of drying nodes and elements
- Performance of renumbering to minimize bandwith
- Realistic representation of Dry/Wet condition
- Reduction of computation time by eliminating drying nodes and elements

Deforming Grid Method

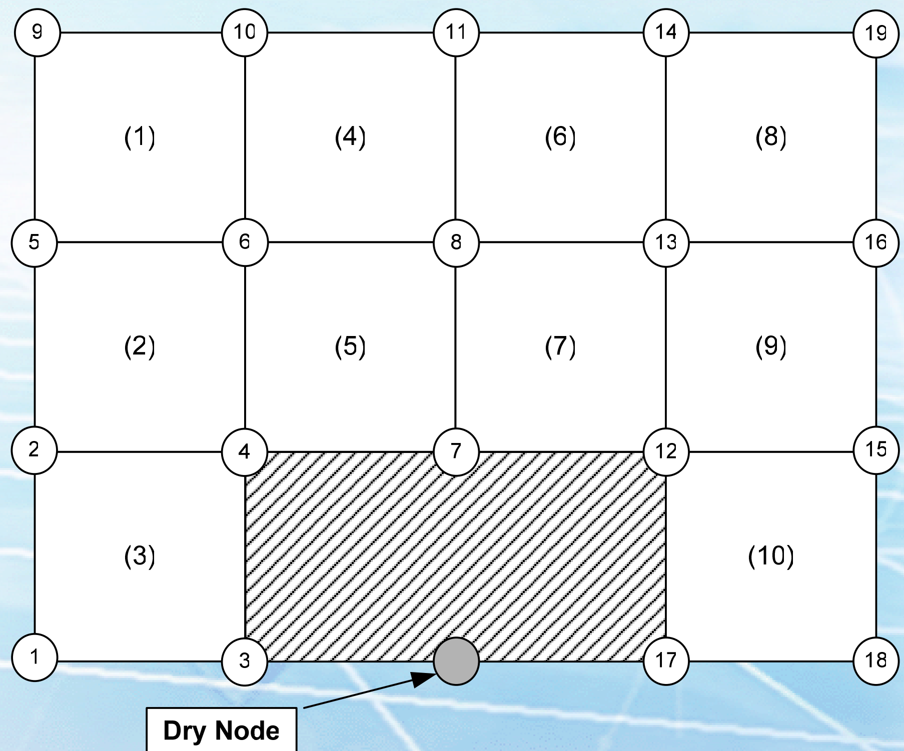


Deforming Grid Method

- Renumbering to minimize bandwith
- Specification of new boundary conditions



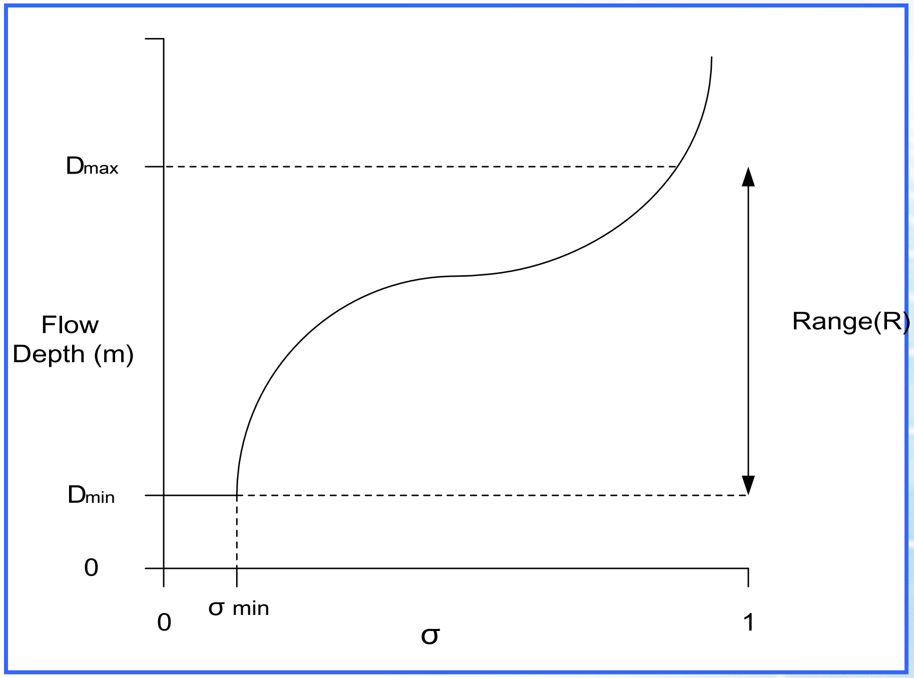
Before Renumbering



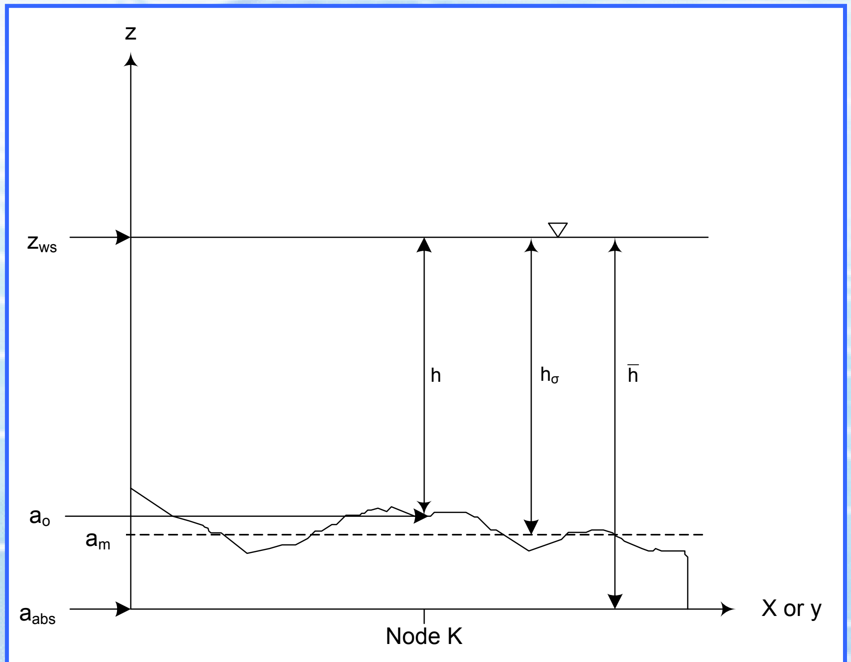
After Renumbering

Transition Element Method

- Method to analyze transition region which exist under dry/wet condition
- Improvement of computation for boundary of wetting and drying area
- Effective computation for flooding area



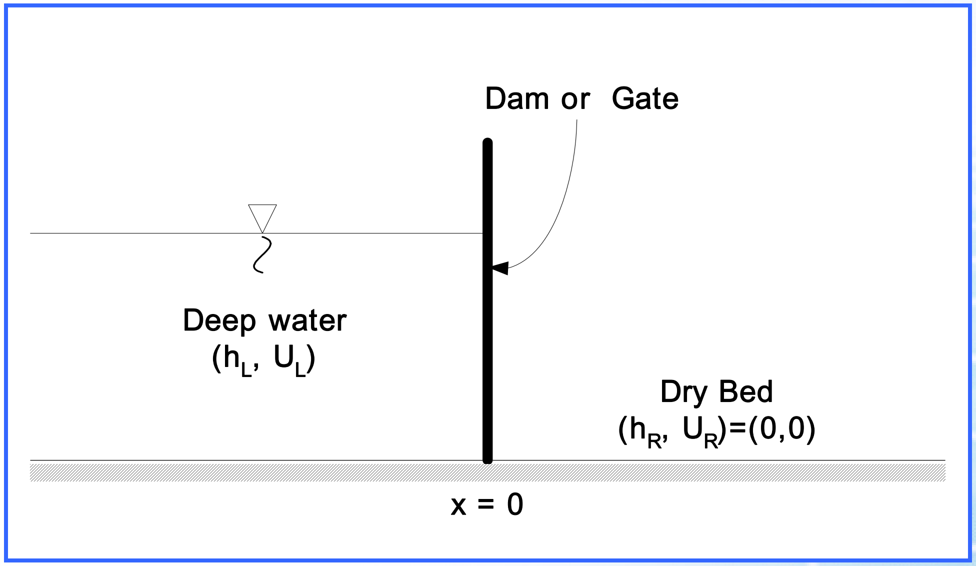
Concept of Domain Coefficient, σ



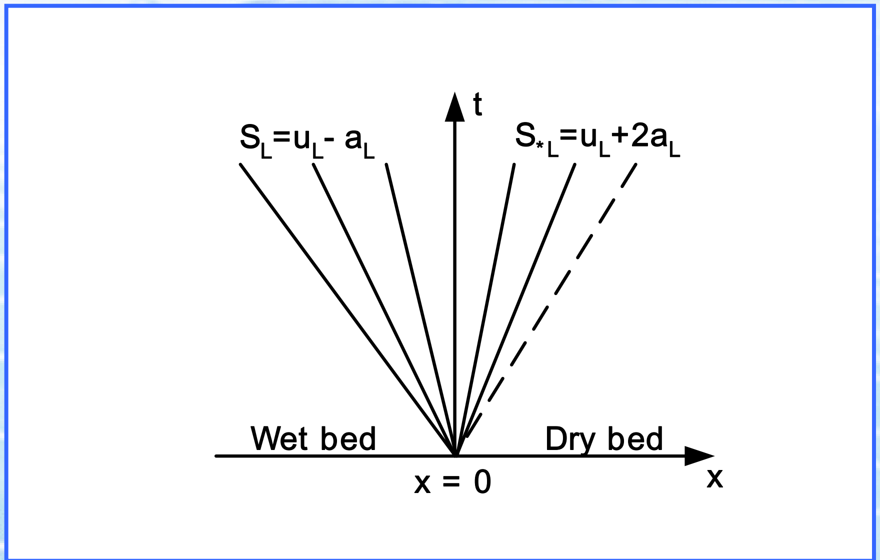
Vertical section through a control volume of the flow domain

Hybrid method (FE and FV method)

- Numerical problems which produce under Dry Bed
  - Difficulty of numerical computation for flow interchange between dry and wet bed

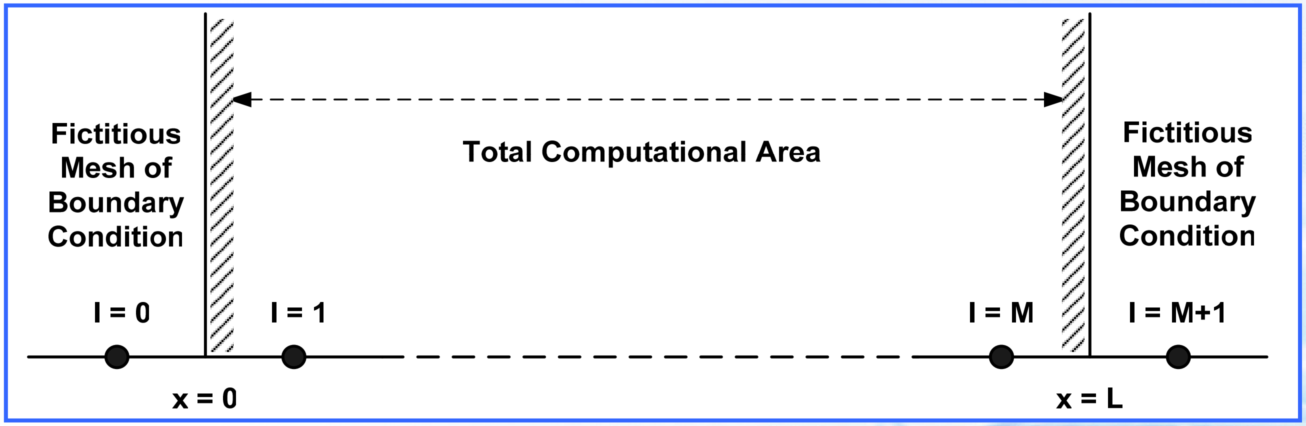


Riemann problem in dry bed

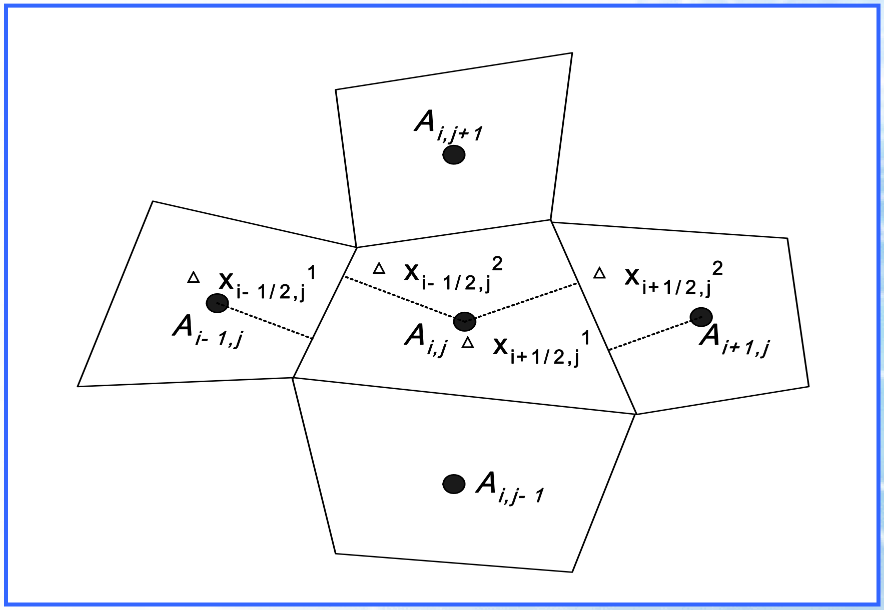


Solutions w/ dry bed

Hybrid method

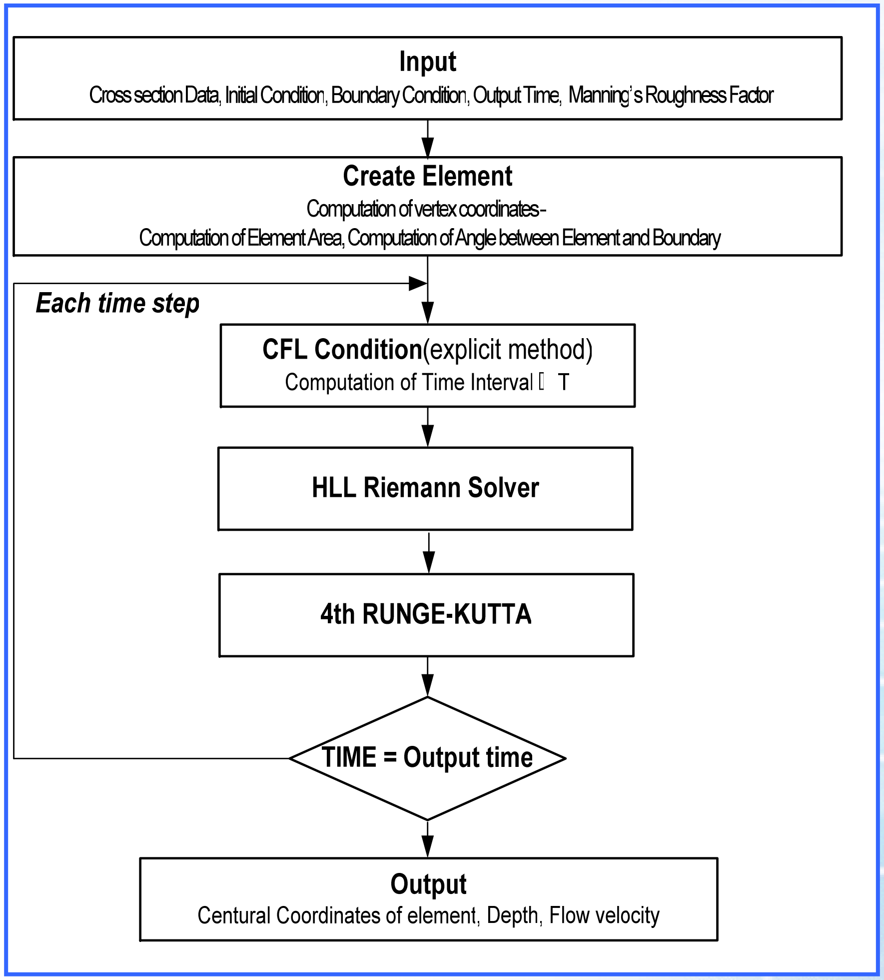


Fictitious Mesh in Computation Area

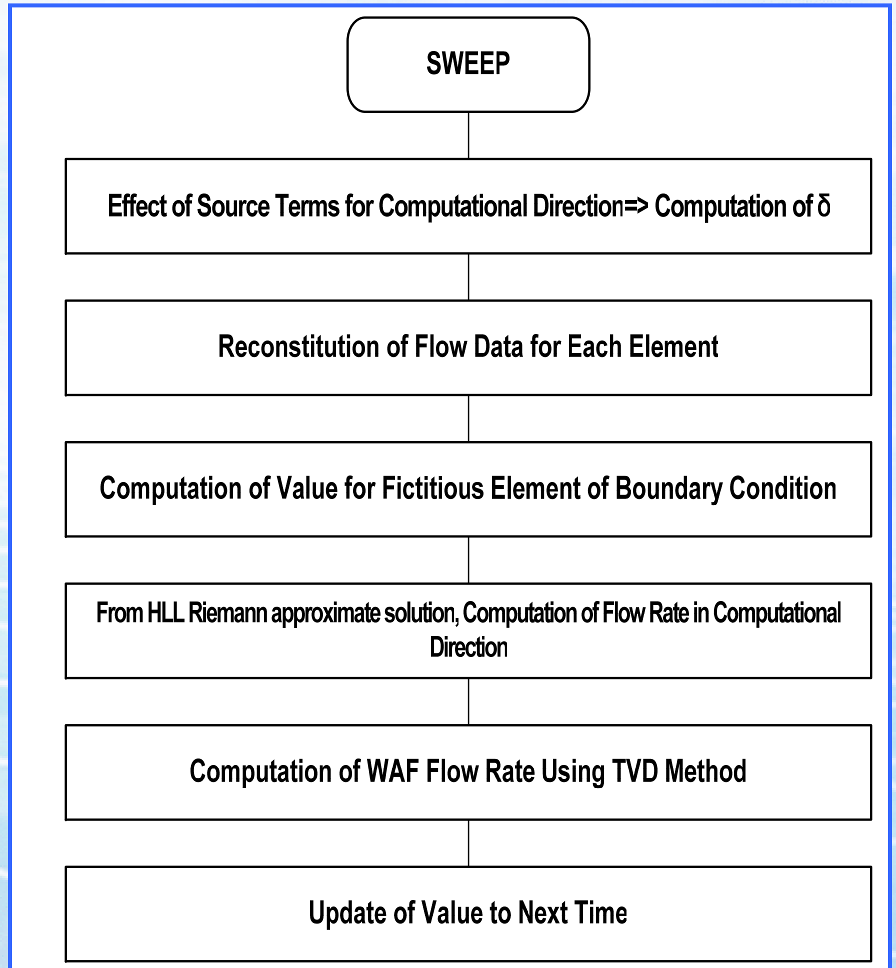


Mesh Structure

Hybrid method

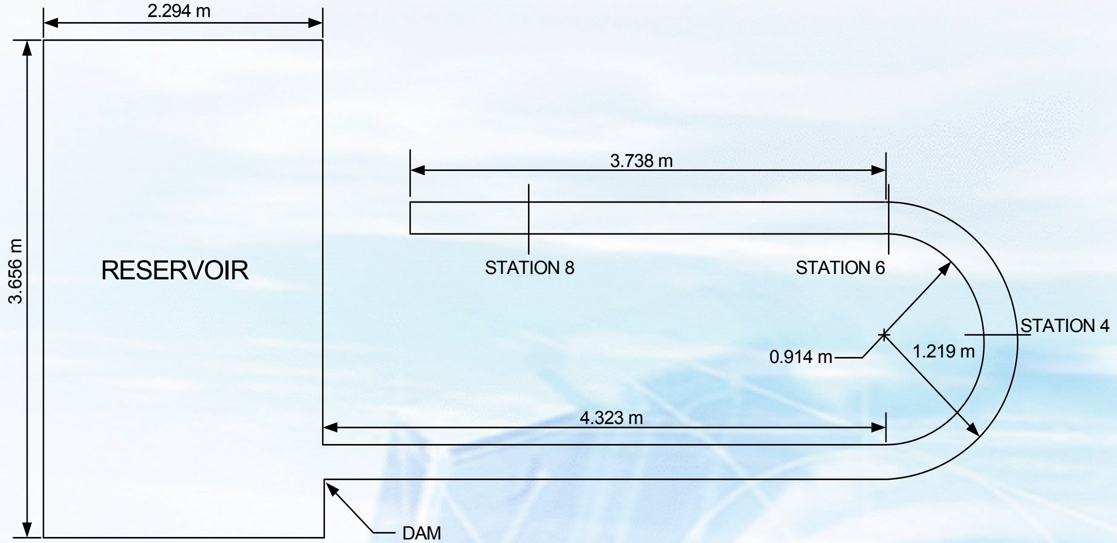


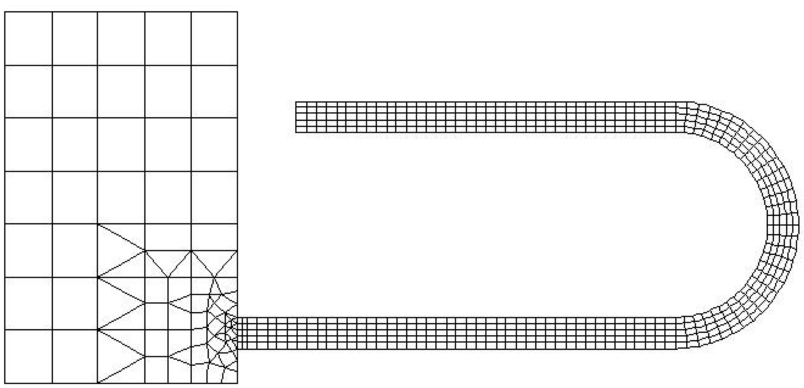
Computation Flowchart

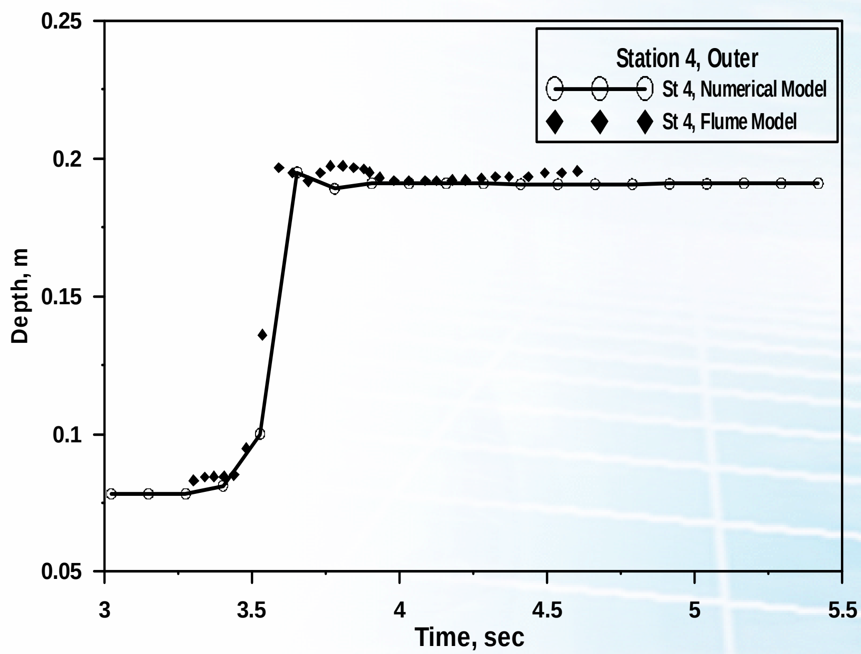


Computation of flow variables by using HLL Riemann approximation solution

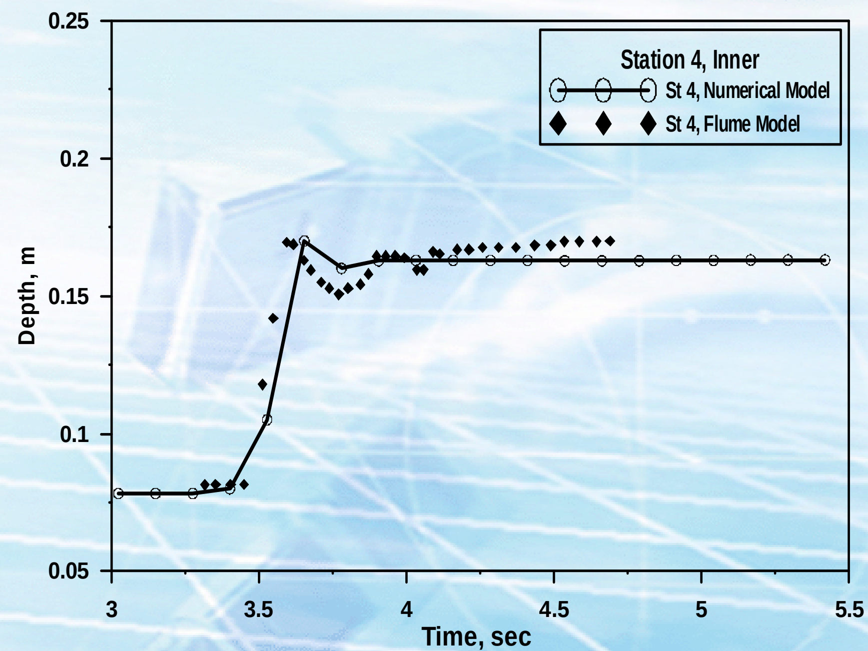
Application to U-shaped Channel



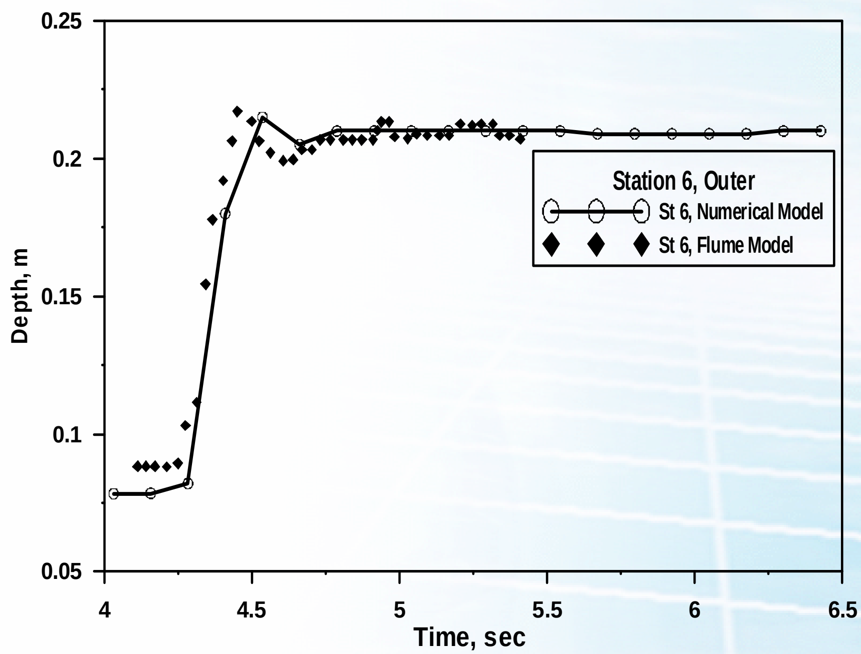




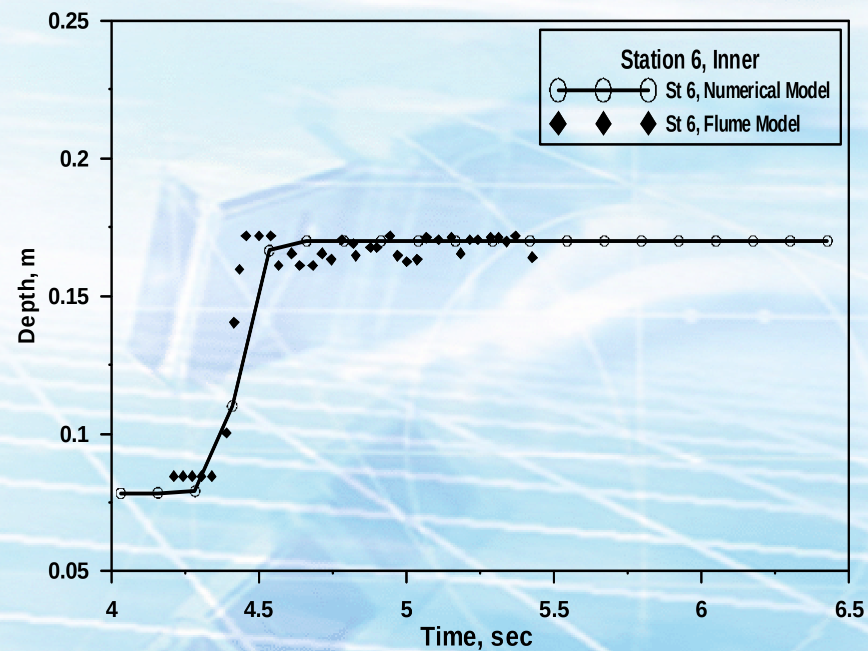
(a) Station 4 (outer side)



(b) Station 4 (inner side)

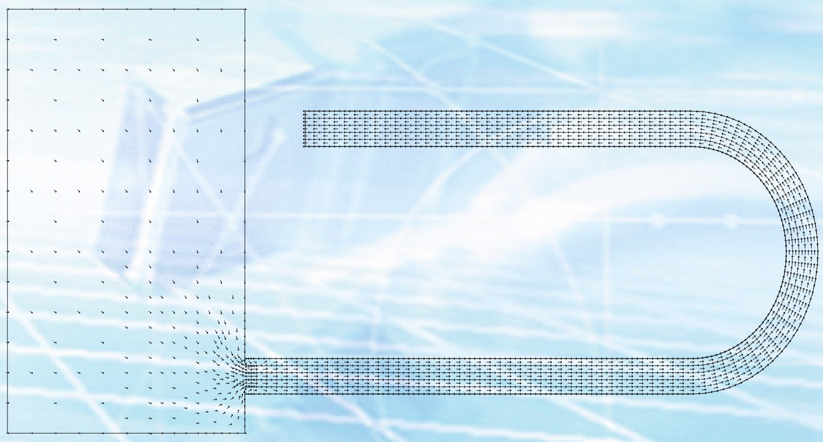


(a) Station 6 (outer side)



(b) Station 6 (inner side)

(a) t = 7.56s

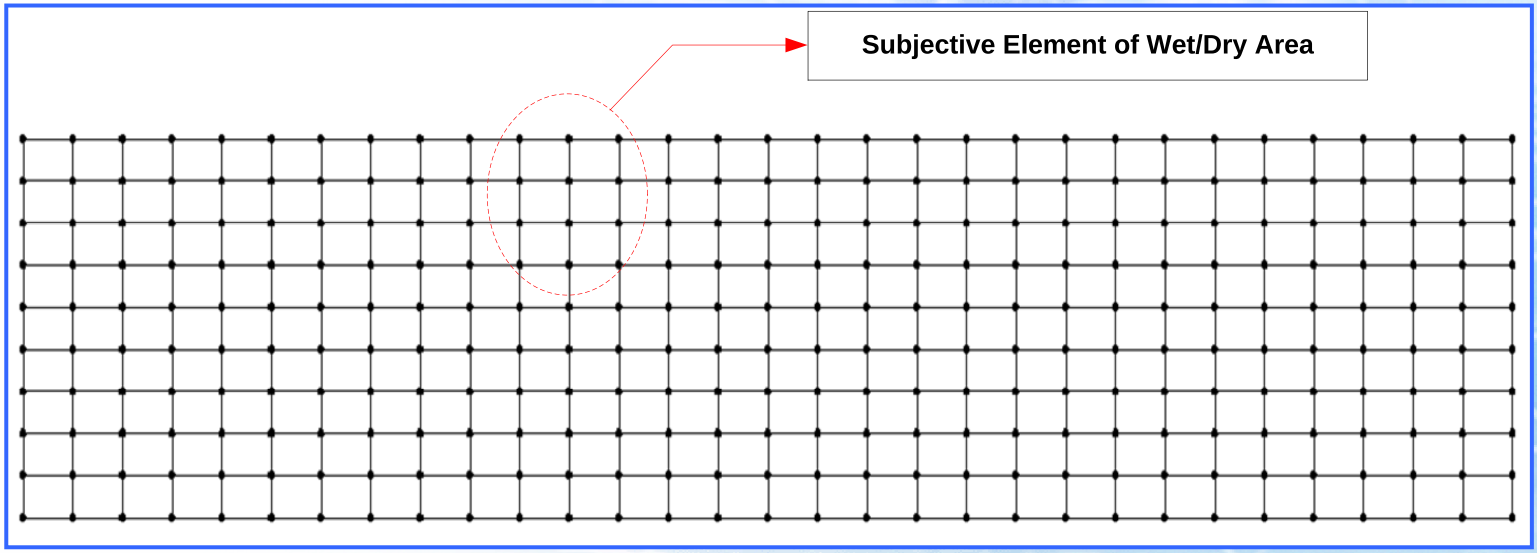


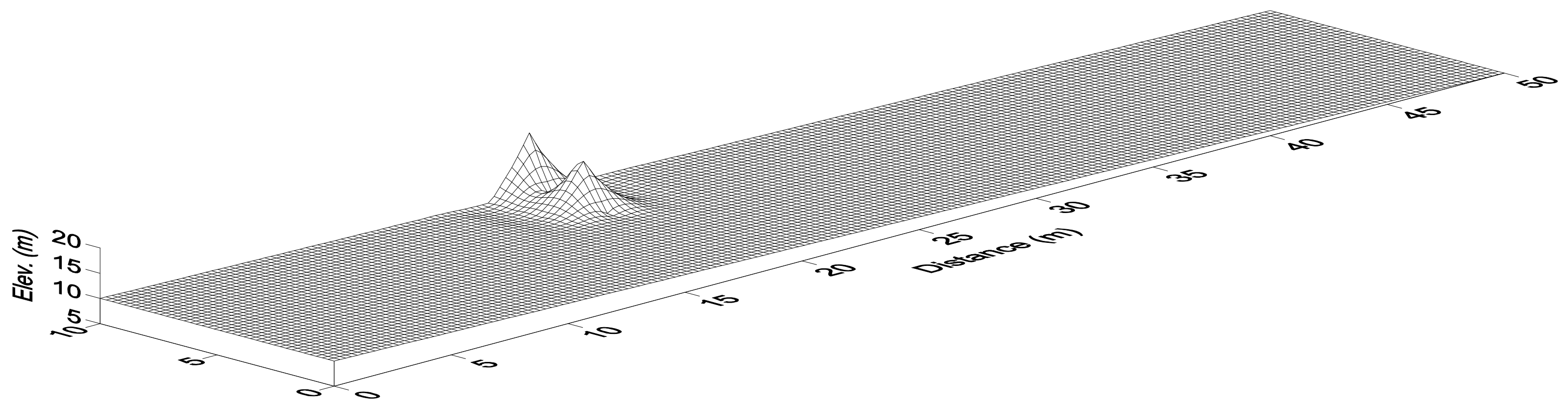
(b) t = 10.08s

Velocity Distributions

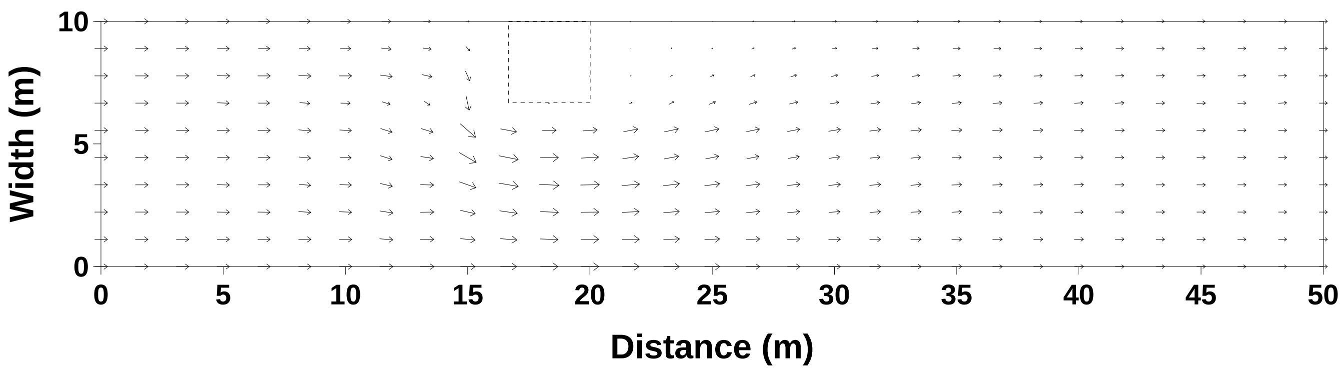
Simple Channel with Dry Bed

- Size of simple channel
  - Length : 50m
  - Bottom slope : 0.1
  - Number of node : 310, Number of element : 270
- Computation Time
  - Time step : 0.01hr
  - Total computation time : 1hr





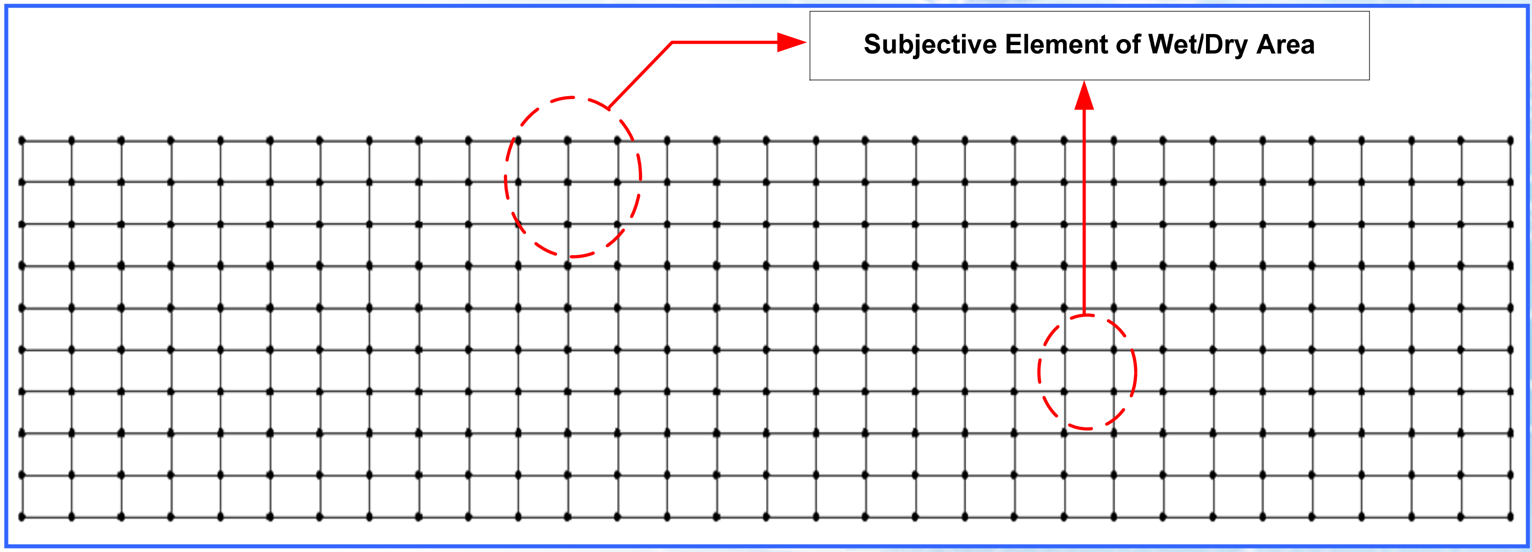
Bottom elevation

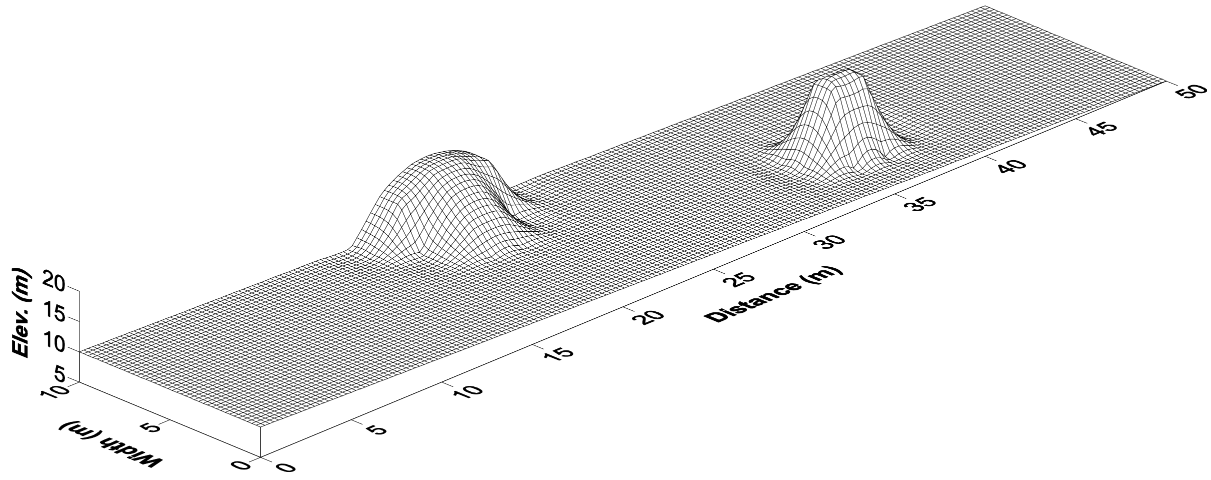


Velocity distribution

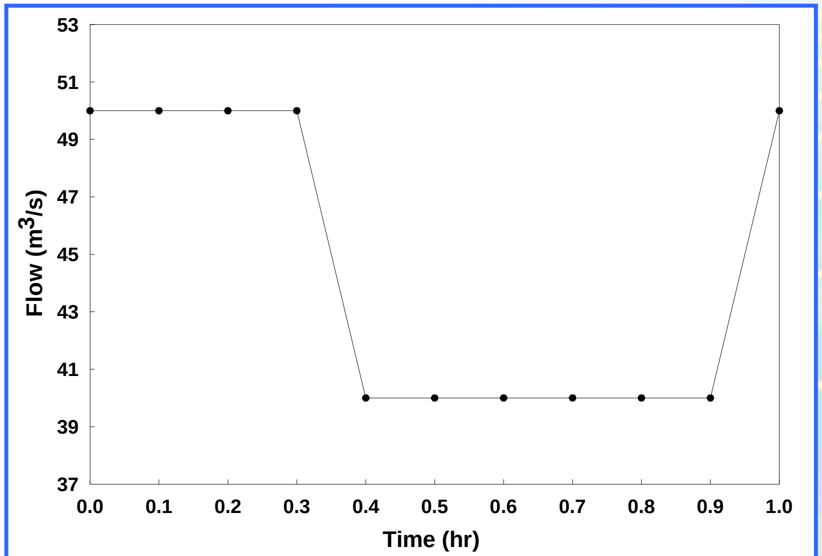
Applications to Simple Channel

- Size of simple channel
  - Length : 50m
  - Bottom slope : 0.1
  - Number of node : 310, Number of element : 270
- Computation Time
  - Time step : 0.01hr
  - Total computational time : 1hr

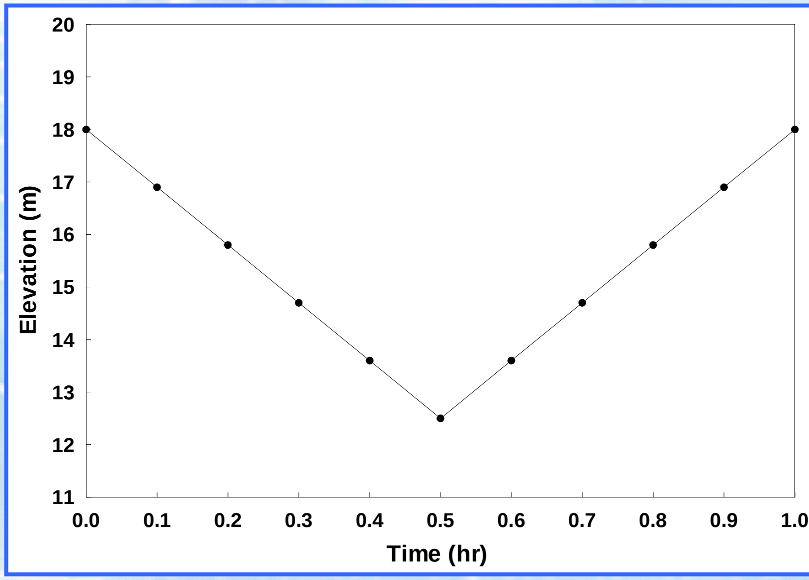




Bottom elevation

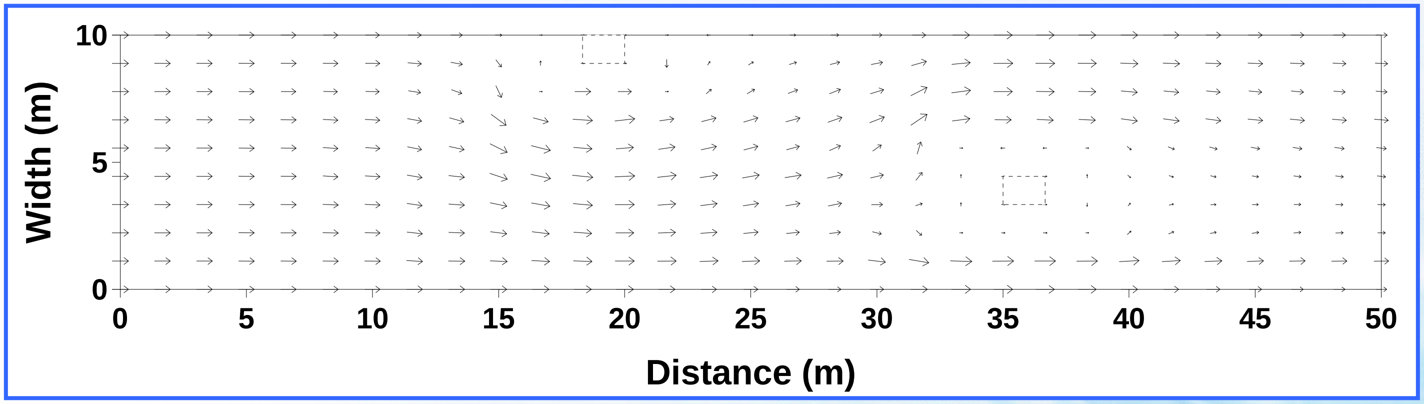


U/S Boundary Condition

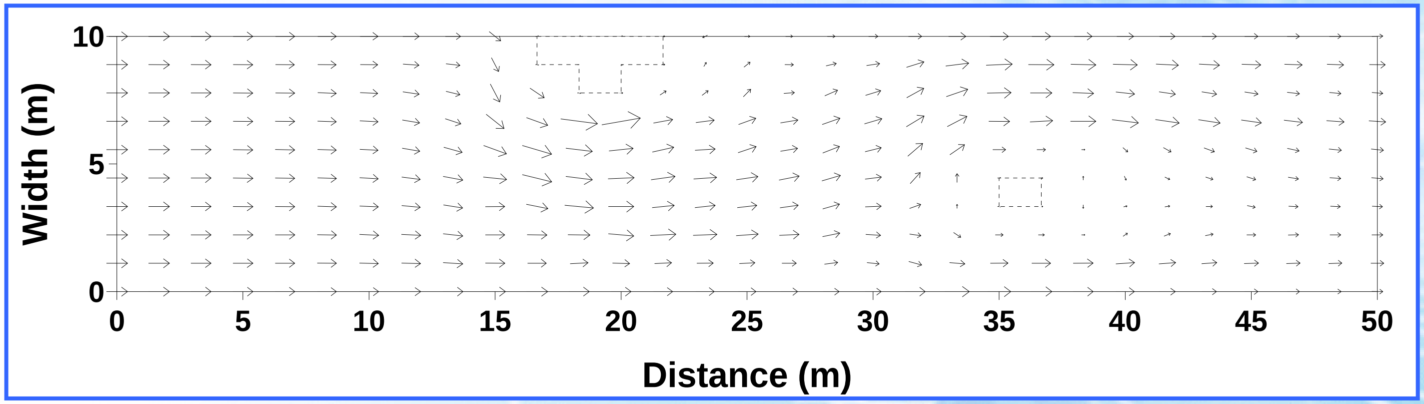


D/S Boundary Condition

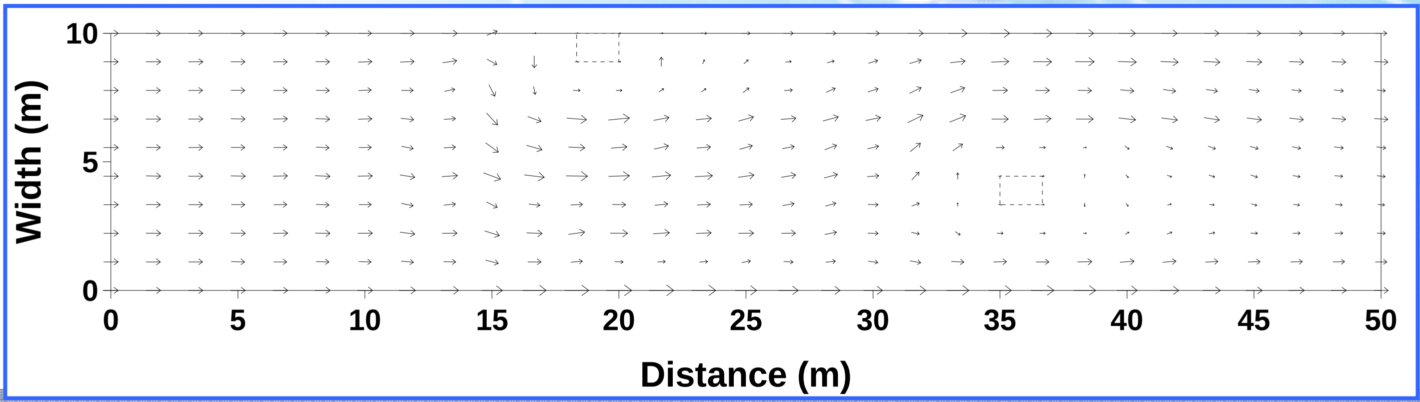
0.2 hr



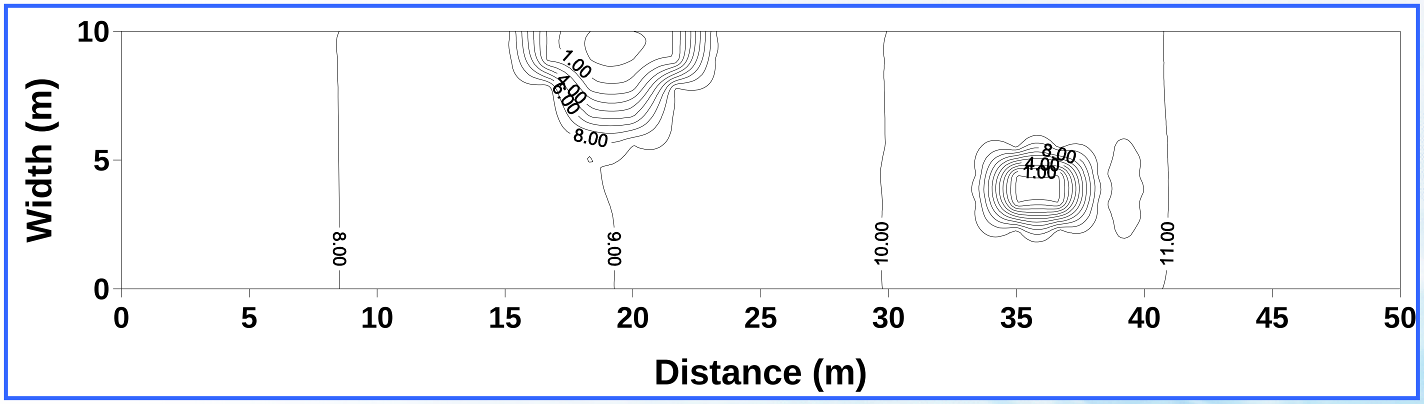
0.4 hr



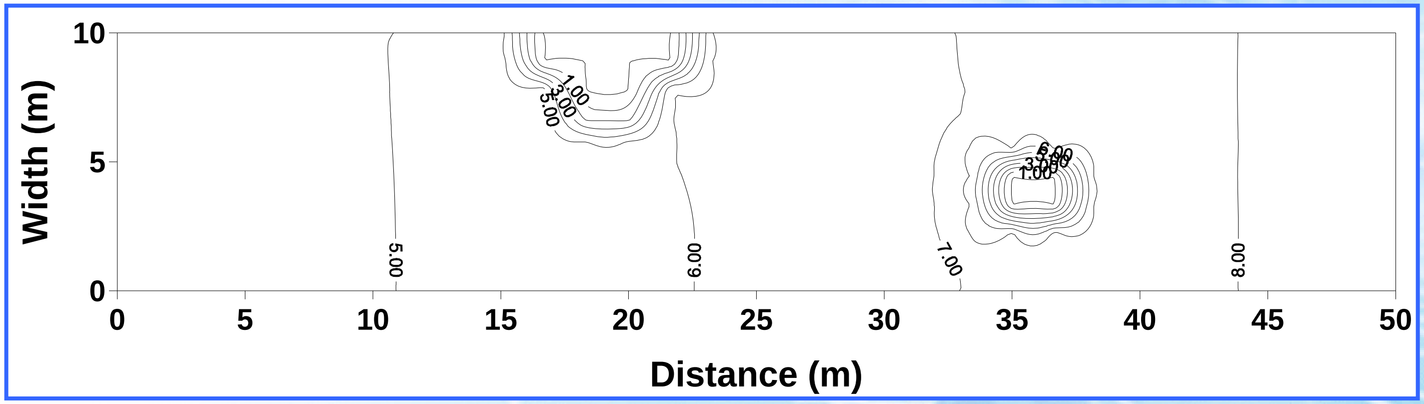
1.0 hr



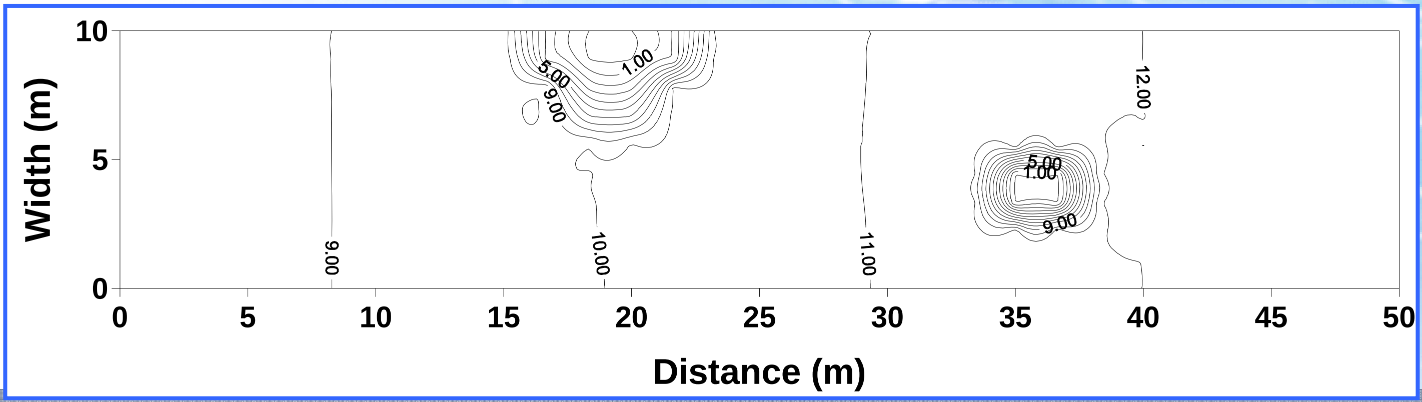
0.2 hr



0.4 hr

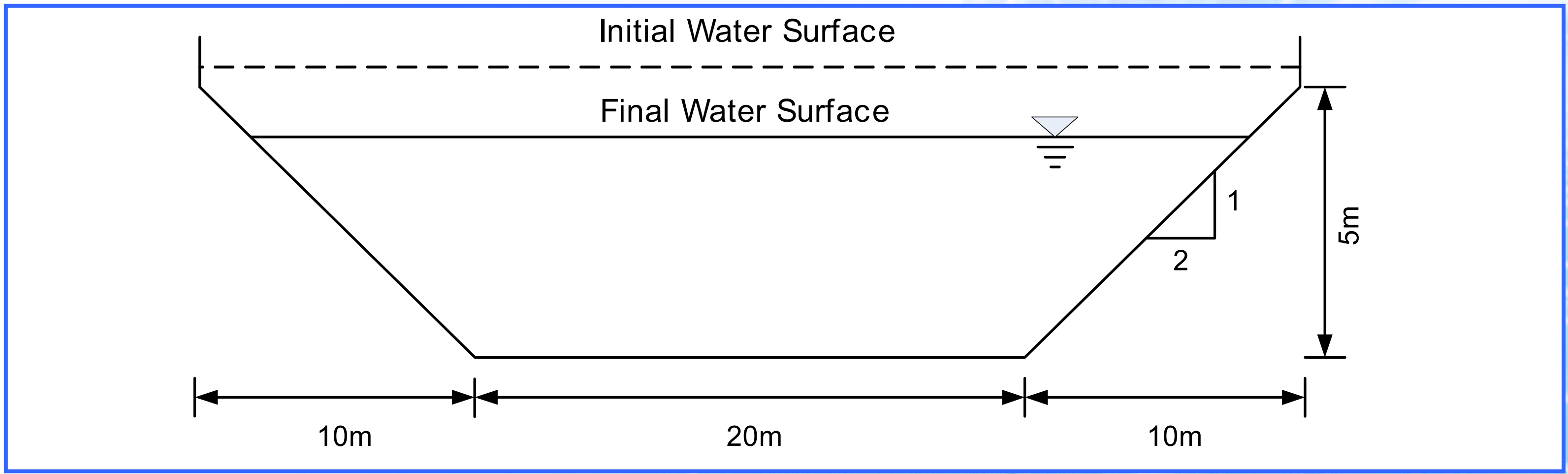


1.0 hr

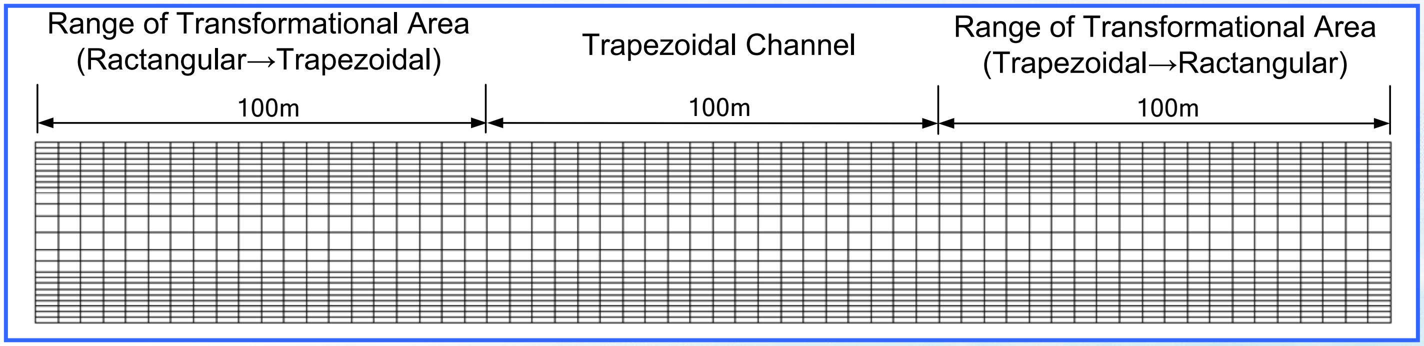


Trapezoidal Channels (partly dry side slope)

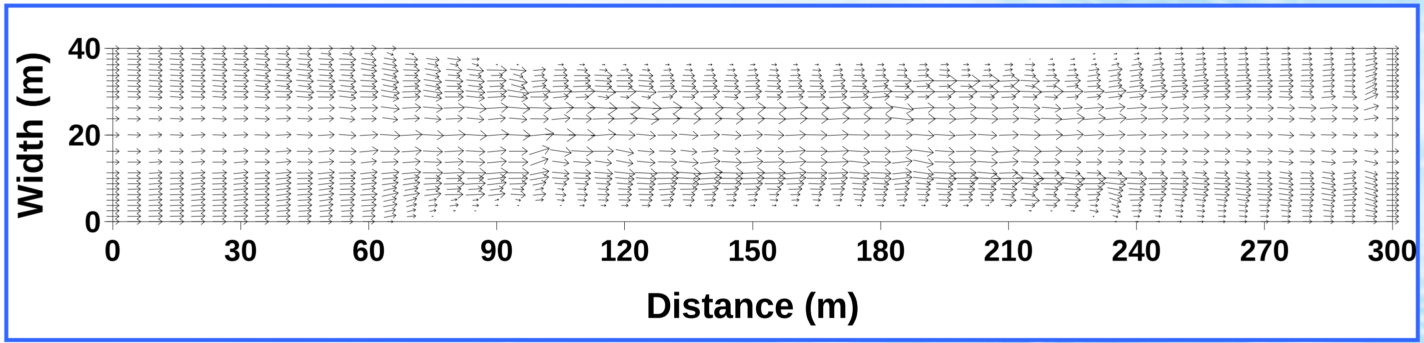
- Size of simple channel
  - Length : 300m, Width : 40m
  - Channel with trapezoidal cross section
- Boundary Condition
  - U/S Boundary condition : 80m^3/s
  - D/S Boundary condition : 4.0m



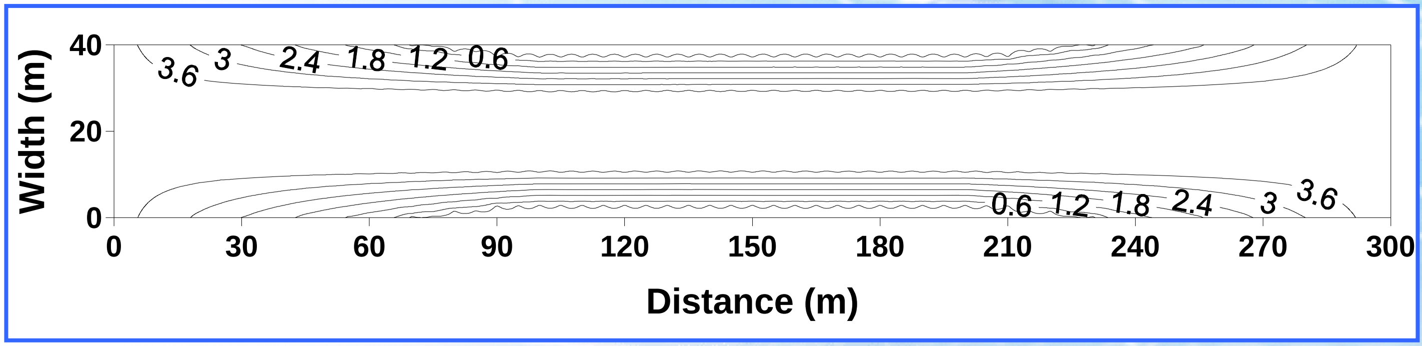
Trapezoidal cross section



Construction of finite element mesh

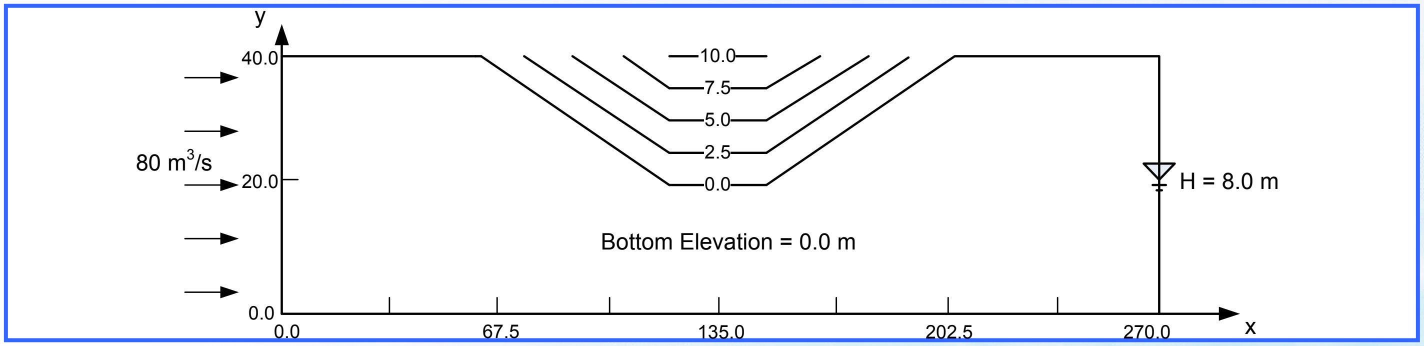


Velocity distribution

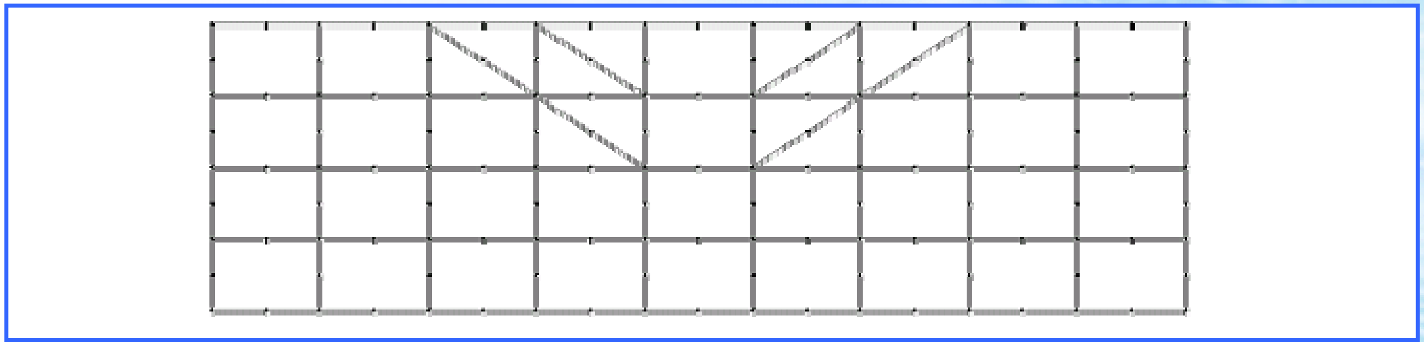


Depth contour

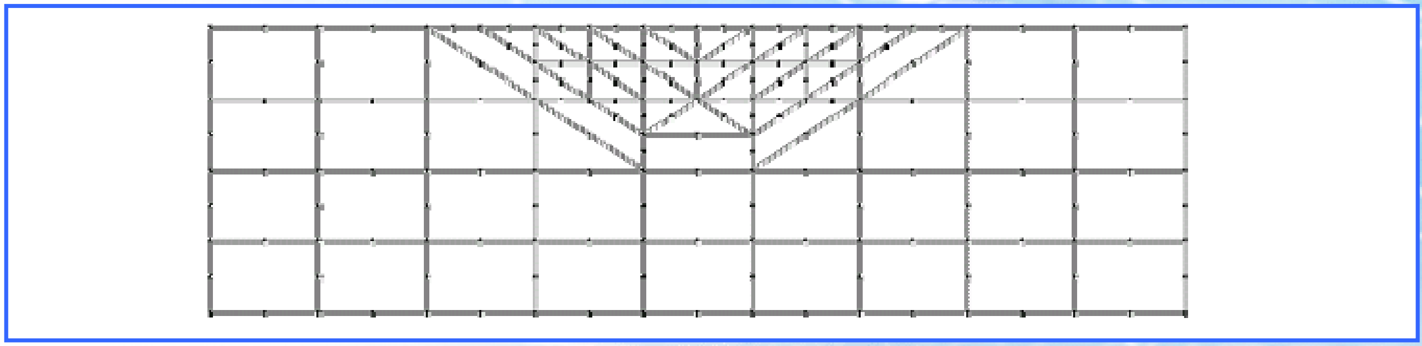
Mesh Resolution Effects with Dry Bed



Channel

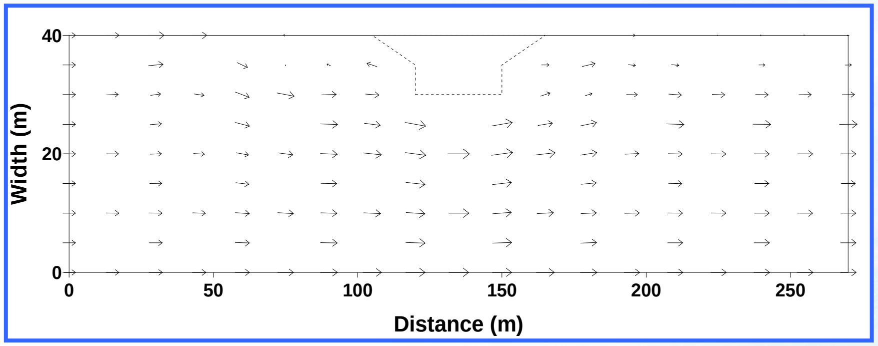


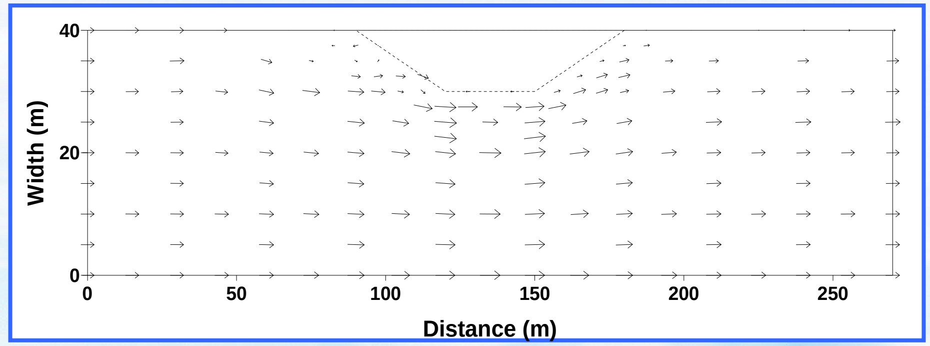
coarse element



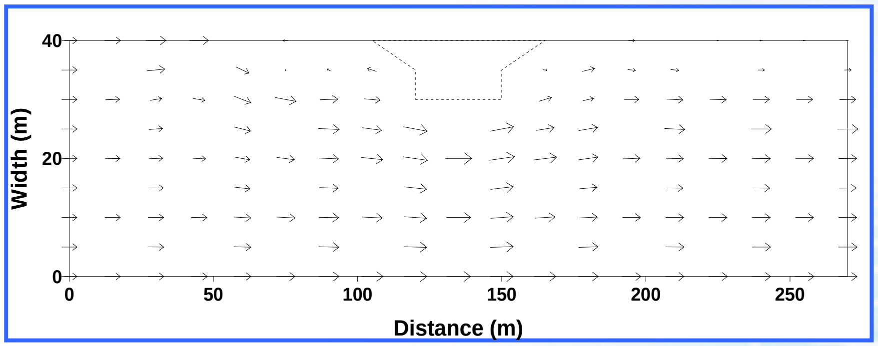
refined element

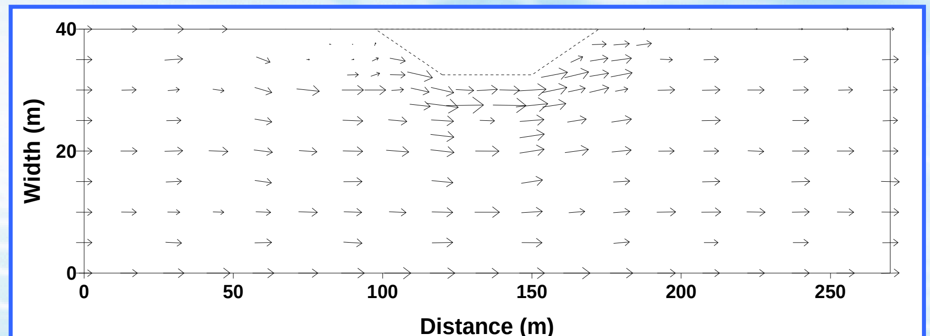
60 m/s^3



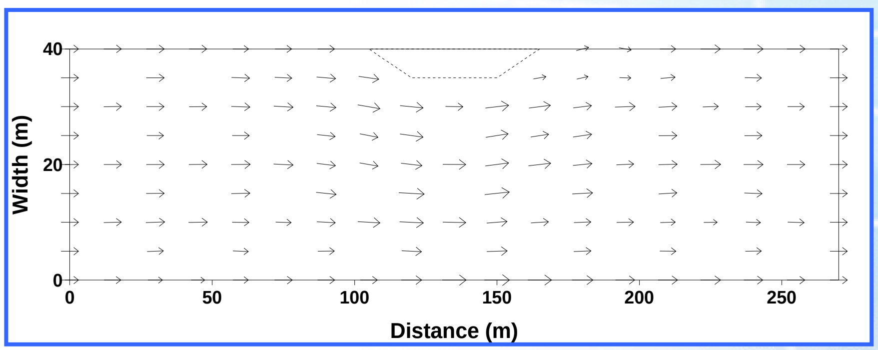


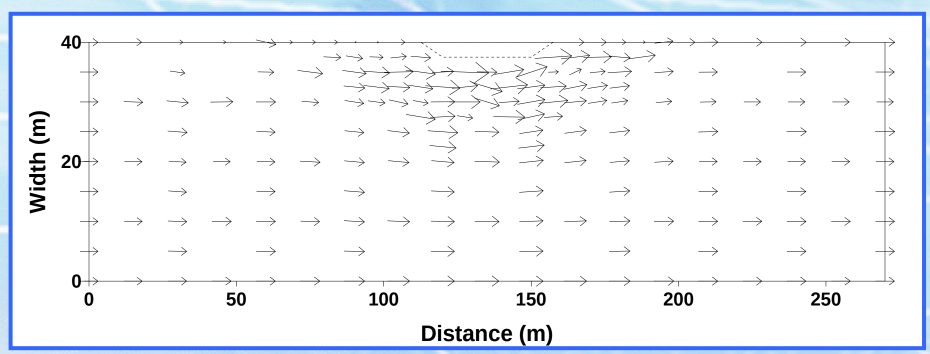
80 m/s^3





120 m/s^3



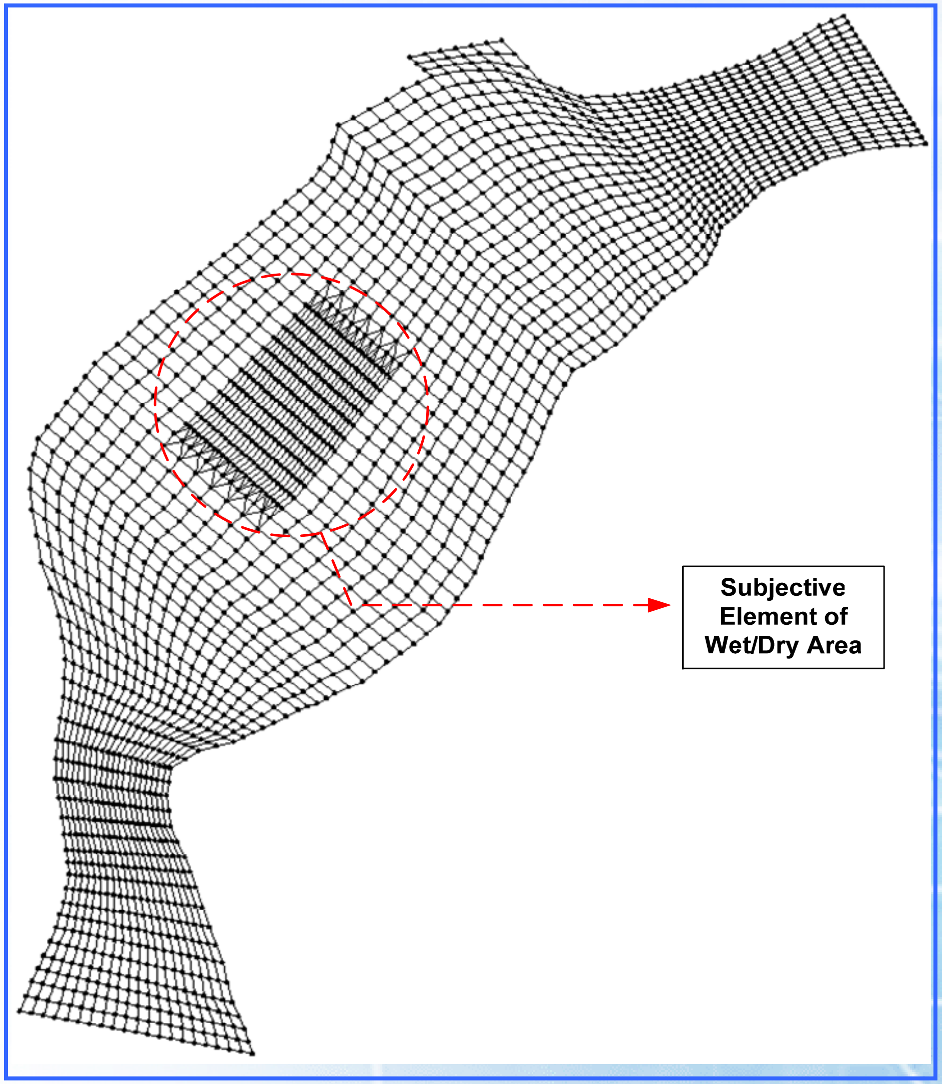


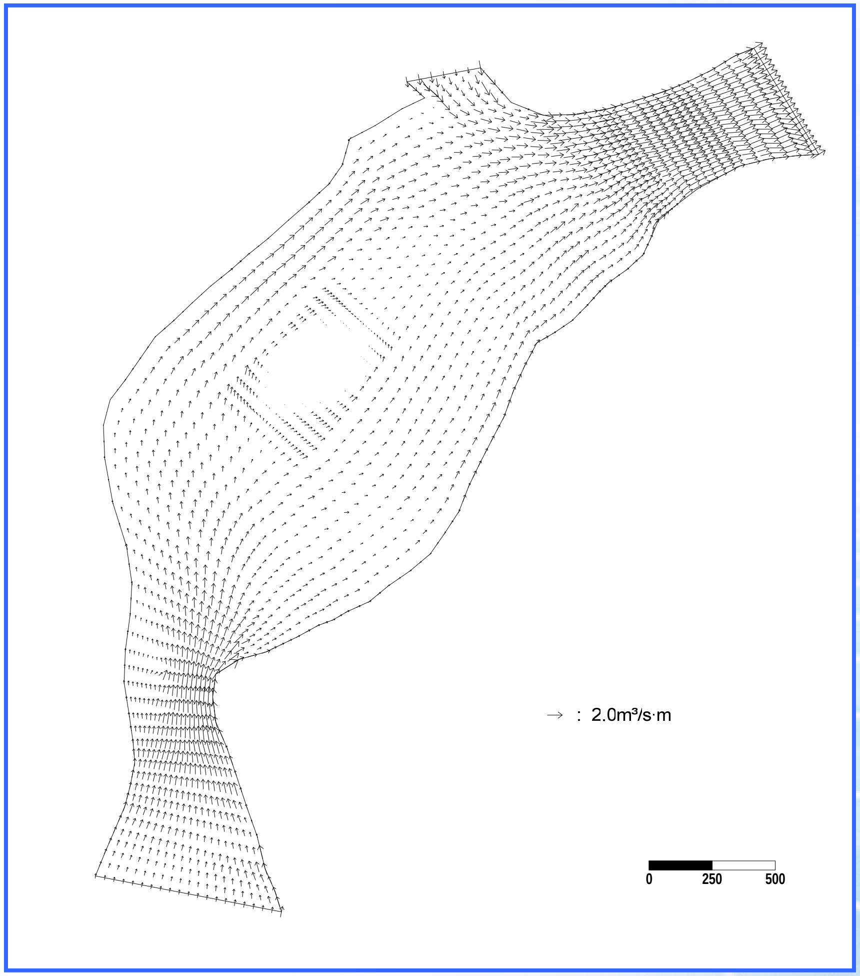
coarse element

refined element

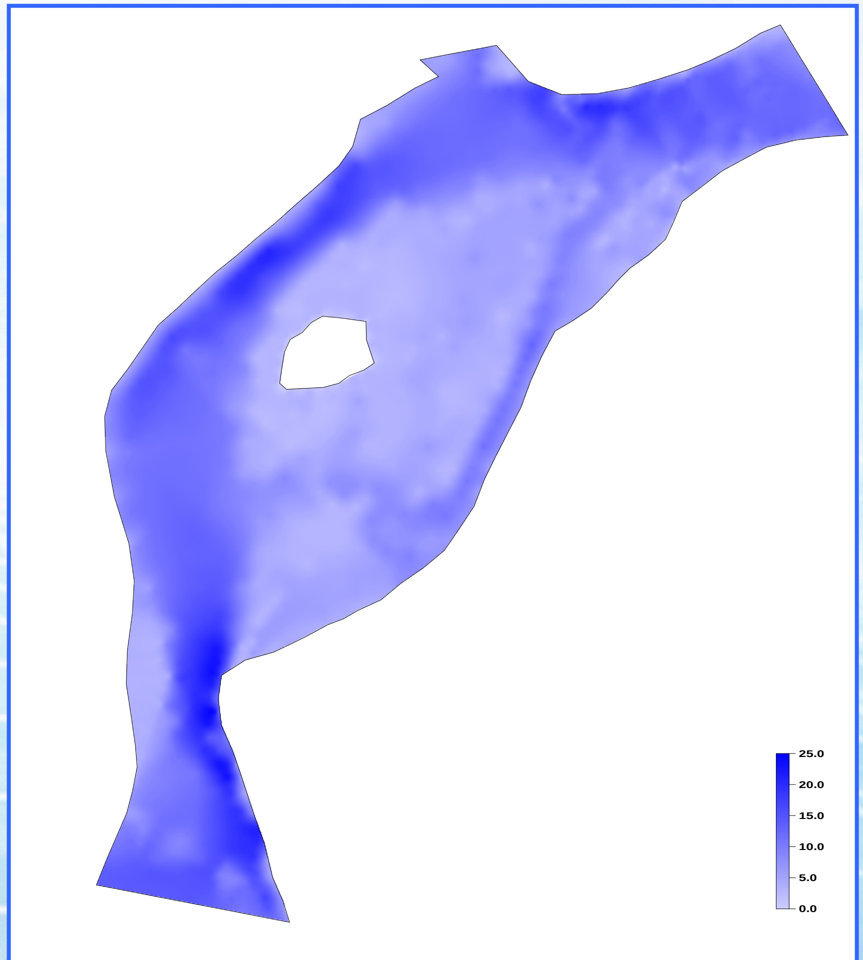
Application to Milyang-river

Milyang-river


Subjective
Element of
Wet/Dry Area

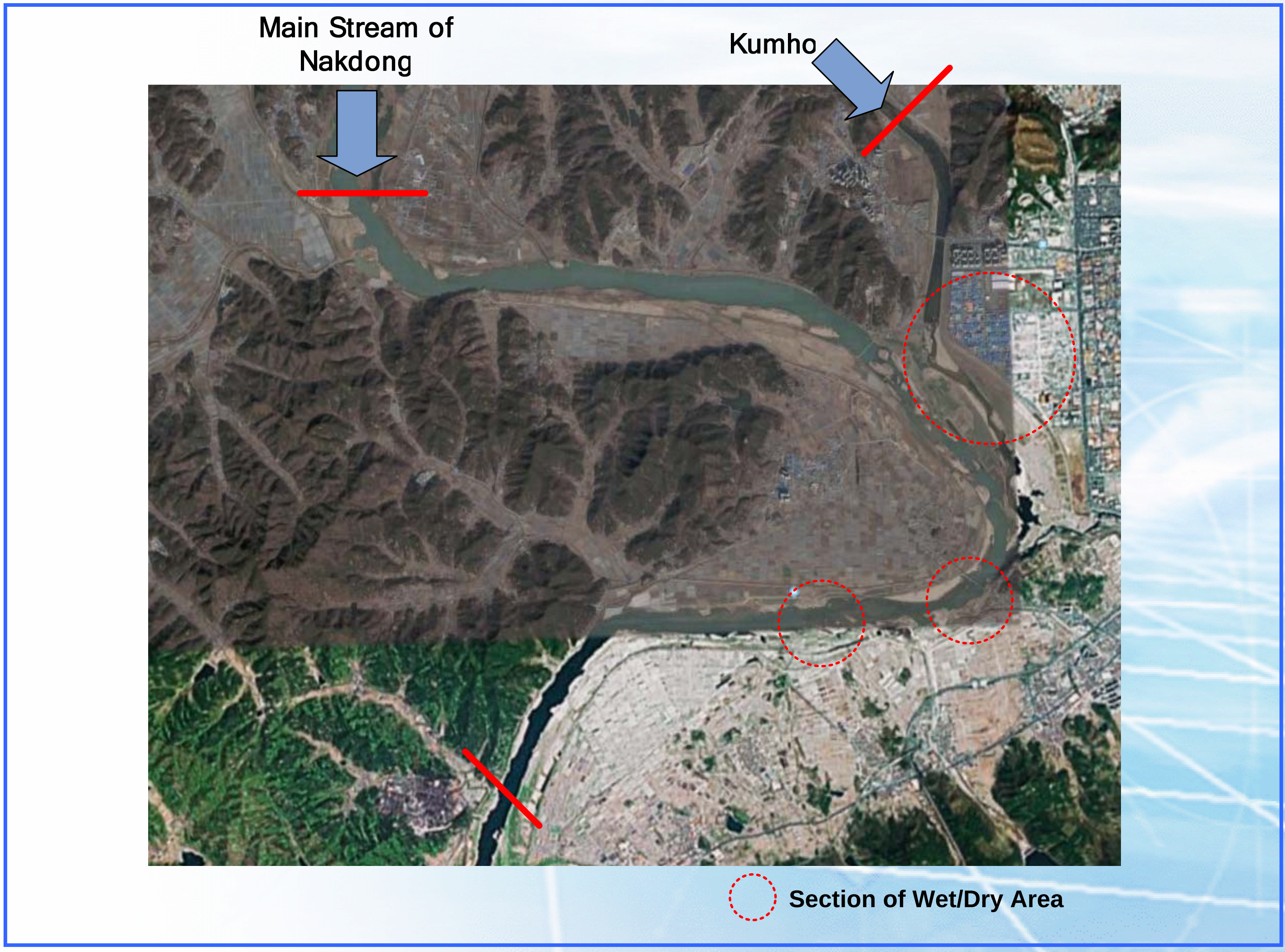


Velocity distribution

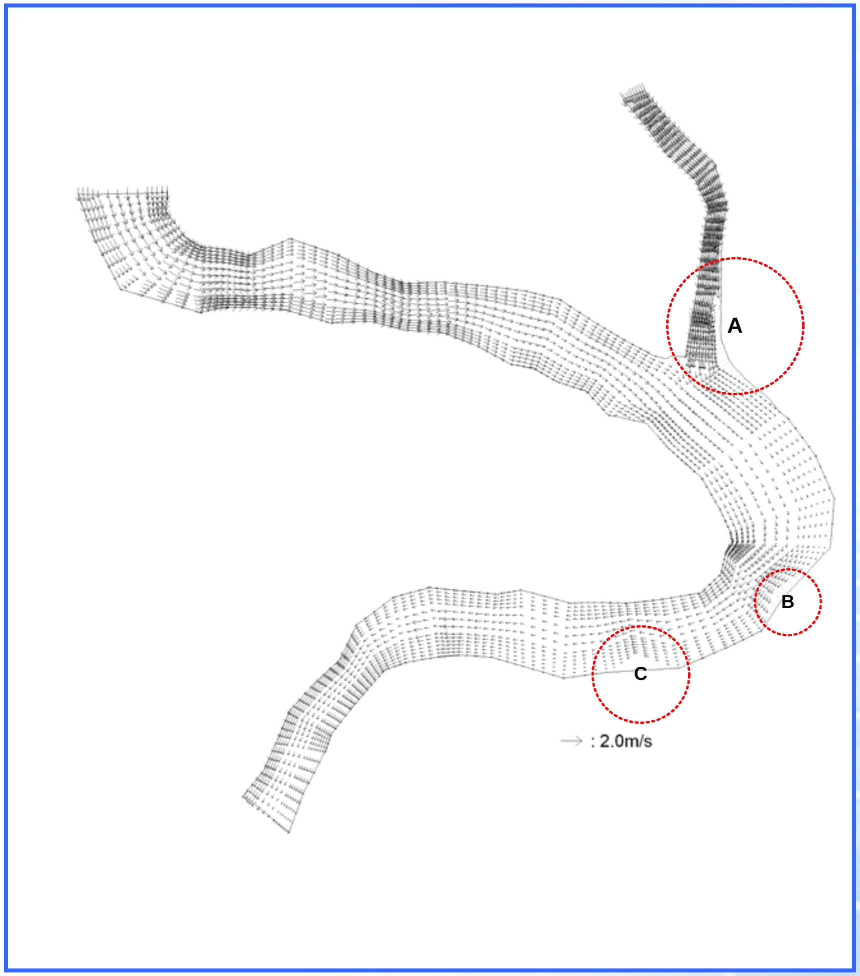


Water surface elevation

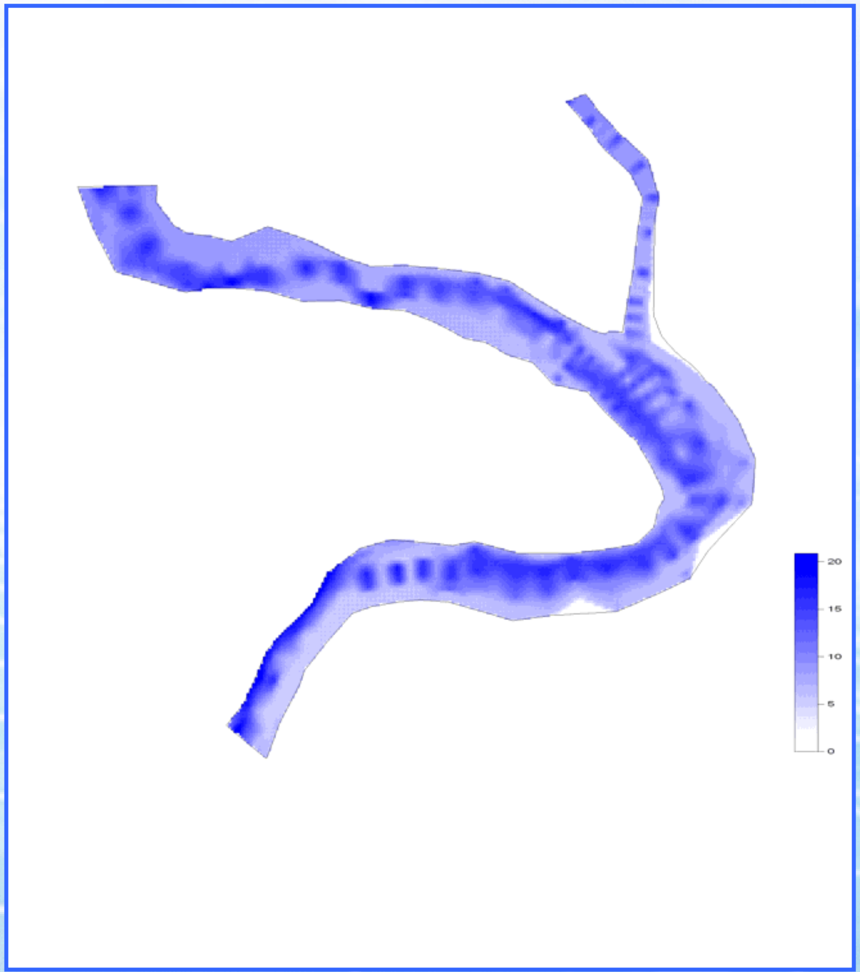
Application to Nakdong-river



Nakdong-river



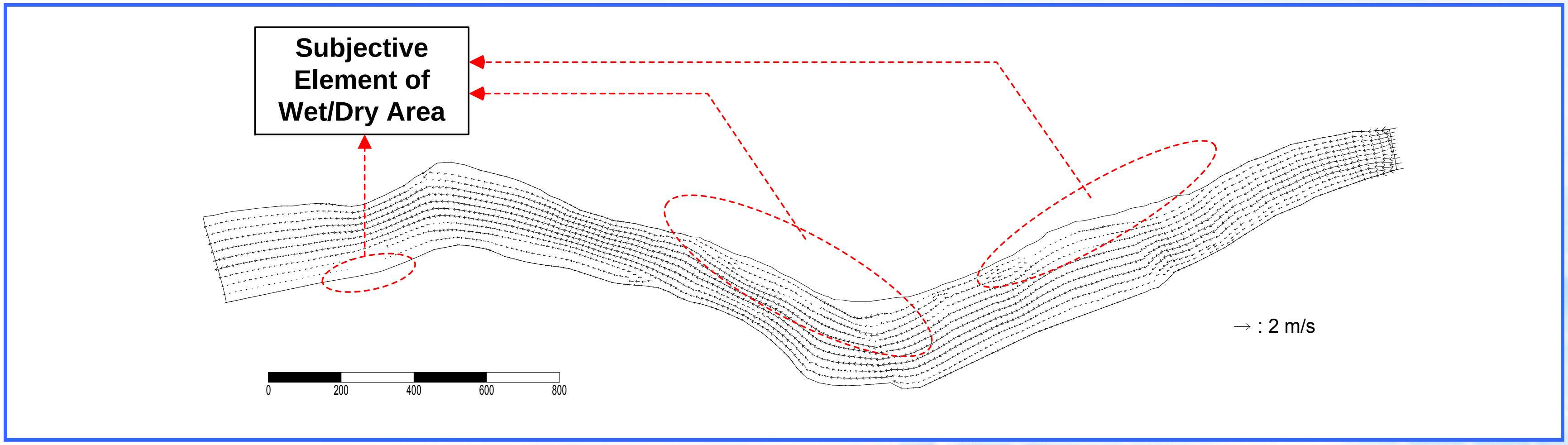
Velocity distribution

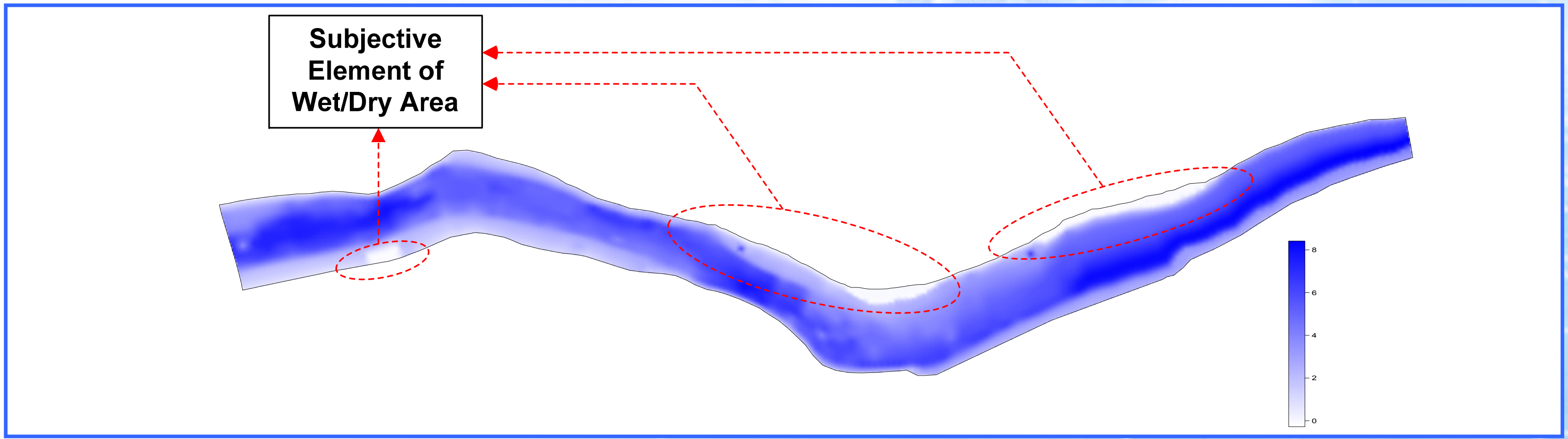


Water surface elevation

Application to Keum River

Kum-river





Velocity distribution and Water surface elevation

Conclusions(1)

(1) The challenging problem facing two-dimensional model is the treatment of wet and dry areas. This situation is encountered in most practical river and coastal engineering problems.

(2) To solve the dry/wet problems, deforming grid method, transition element method, and hybrid method are adopted.

(3) The model is verified by applying to U-shaped laboratory channel. The simulation result agree with observed data for various flow conditions.

Conclusions(2)

(4) Trapezoidal channel with partly dry side slopes, straight channel with various drying conditions, are examined for flow model validations.

(5) RAM2 model shows reasonable flow distribution compared with existing model in dry area simulation in Milyang-river, Nakdong-river and Keum-river.

(6) RAM2 model can be used as a hydrodynamic input for water quality and sediment transport model as well as be used for analyzing dam-break flow, levee-break flow and rapidly varied flow.

Thank You ..