Modeling of 2D Flood Flow Analysis
by Finite Element Method

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Outline

1. Introduction
2. Development of RAM2 Model
3. Model Calibration and Verification
4. Applications of RAM2 to Natural Rivers
5. Conclusions
Introductions

- 2D hydrodynamic model can provide good estimates of complex features in the flow around islands and obstructions, flow at confluence and flow in braided channel.

- The challenging problem facing two-dimensional FE model is the treatment of wet and dry areas. This situation is encountered in most practical river such as flood propagation, dam break analysis, tidal processes and so on.
Objectives of the Study

To develop an accurate and robust two-dimensional finite element method with wet and dry simulation in complex natural rivers.
Scope of the Study

Development of Dry/Wet Algorithm
- Development of Deforming Grid Method
- Development of Transition Element Method
- Development of Hybrid Method

Application of Dry/Wet Algorithm
- Laboratory U-shaped Channels
- 2D Dambreak-wave, Several cases with dry state

Application of RAM2 Model to natural channel
- Application of RAM2 Model for Milyang-river, Nakdong-river, Kum-river
Watershed Analysis in Hydrology/Hydraulic Aspect

Analysis Model of Hydrology in Watershed

Numerical River Analysis Model adopted Wet/Dry Treatment

Deforming Grid Method  Transition Element Method  Hybrid Method

Numerical Analysis Results

Analysis of River Convection-Diffusion  Analysis of River Bed Variation

Connected with GUI

α-test and β-test

River Model Development

Estimation of Water Budget and Mass Budget

Practical Use of Sustainable Water Resources Research Establishment
Governing Equation (Matrix Form)

\[
\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} + D = 0
\]

\[
\begin{align*}
U &= (h, p, q)^T \\
F &= \begin{pmatrix} \frac{p^2}{h} + \frac{gh^2}{2} \\
\frac{pq}{h} \\
\frac{pq}{h} \end{pmatrix} = M_x U = \begin{pmatrix} 0 \\
\frac{gh}{2} \\
0 \\
u \\
0 \\
u \end{pmatrix} U \quad G = \begin{pmatrix} q \\
\frac{pq}{h} \\
q^2 + \frac{gh^2}{2} \end{pmatrix} = M_y U = \begin{pmatrix} 0 \\
0 \\
v \\
0 \\
v \end{pmatrix} U \\
D &= \begin{pmatrix} gh \frac{\partial z^0}{\partial x} + gn^2 \frac{p(p^2 + q^2)^{1/2}}{h^{7/3}} - \frac{i}{\partial x} (\varepsilon_{xx} \frac{\partial p}{\partial x}) - \frac{i}{\partial y} (\varepsilon_{xy} \frac{\partial p}{\partial y}) - \frac{p}{h} (\varepsilon_{xx} \frac{i}{2}) \\
gh \frac{\partial z^0}{\partial y} + gn^2 \frac{q(p^2 + q^2)^{1/2}}{h^{7/3}} - \frac{i}{\partial x} (\varepsilon_{yx} \frac{\partial q}{\partial x}) - \frac{i}{\partial y} (\varepsilon_{yy} \frac{\partial q}{\partial y}) - \frac{q}{h} (\varepsilon_{yx} \frac{i}{2}) \end{pmatrix}
\end{align*}
\]
Weighting Functions

\[ \int_{\Omega} B_i^T \left( \frac{\partial \hat{U}}{\partial \tau} - \frac{\partial x}{\partial \tau} \frac{\partial \hat{U}}{\partial x} - \frac{\partial y}{\partial \tau} \frac{\partial \hat{U}}{\partial y} + A \frac{\partial \hat{U}}{\partial x} + B \frac{\partial \hat{U}}{\partial y} + D \right) d\Omega = 0 \]

Bubnov-Galerkin Scheme

\[ [B_i'] = [B_i] \]

Petrov-Galerkin Scheme

\[ [B_i'] = [B_i] + \varepsilon \left[ \begin{array}{c} A \frac{dB_i}{dx} \\ B_i \end{array} \right] + \varepsilon \left[ \begin{array}{c} A \frac{dB_i}{dx} \\ B_i \end{array} \right] \]

\[ \varepsilon_x = \frac{\Delta x}{|U + c| \sqrt{15}} \]

\[ \varepsilon_y = \frac{\Delta y}{|V + c| \sqrt{15}} \]

\[ \varepsilon = \frac{2}{\sqrt{15}} \approx 0.50 \]

SU/PG Scheme

\[ [B_i'] = [B_i] + \alpha \frac{\Delta x}{2} \left[ \begin{array}{c} A \frac{dB_i}{dx} \\ B_i \end{array} \right] + \frac{\Delta y}{2} \left[ \begin{array}{c} A \frac{dB_i}{dx} \\ B_i \end{array} \right] \]

\[ [W_x] = [M_x] \left[ \begin{array}{c} \lambda_i \\ \lambda_i \end{array} \right] [M_x]^{-1} \]

\[ [W_y] = [M_y] \left[ \begin{array}{c} \lambda_i \\ \lambda_i \end{array} \right] [M_y]^{-1} \]
Upwinding Matrix

**Upwinding Matrix of PG Scheme (2D)**

\[
W_x = \begin{bmatrix}
0 & 1 & 0 \\
\frac{c^2 - U^2}{|U + c|} & \frac{2U}{|U + c|} & 0 \\
-UV & V & U \\
|U + c| & |U + c| & |U + c|
\end{bmatrix}
\]

**Upwinding Matrix of SU/PG (2D)**

\[
W_y = \begin{bmatrix}
0 & 0 & 1 \\
\frac{-UV}{|V + c|} & V & |V + c| \\
\frac{c^2 - V^2}{|V + c|} & 0 & 2V \\
|V + c| & |V + c| & |V + c|
\end{bmatrix}
\]

**Upwinding Matrix of SU (2D)**

\[
[W_x] = -\frac{1}{2c} \begin{bmatrix}
1 & 0 & 1 \\
\frac{U - c}{|U - c|} & 0 & 0 \\
\frac{U + c}{|U + c|} & 0 & 0 \\
V & 1 & V
\end{bmatrix}
\]

\[
[W_y] = \frac{1}{2c} \begin{bmatrix}
1 & 0 & 1 \\
\frac{V - c}{|V - c|} & 0 & 0 \\
\frac{V + c}{|V + c|} & 0 & 0 \\
V - c & 1 & V + c
\end{bmatrix}
\]
Implementation of SU/PG Method

Implementation of SU/PG for Governing Equation (Matrix Form)

\[
\begin{bmatrix}
B_i & 0 & 0 \\
0 & B_i & 0 \\
0 & 0 & B_i
\end{bmatrix}
\begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix} + \alpha \Delta x \begin{bmatrix}
W_x
\end{bmatrix} \frac{\partial}{\partial x}
\begin{bmatrix}
B_i & 0 & 0 \\
0 & B_i & 0 \\
0 & 0 & B_i
\end{bmatrix}
\begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix} + \alpha \Delta y \begin{bmatrix}
W_y
\end{bmatrix} \frac{\partial}{\partial y}
\begin{bmatrix}
B_i & 0 & 0 \\
0 & B_i & 0 \\
0 & 0 & B_i
\end{bmatrix}
\begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix} = \{0\}
\]

Implementation of SU/PG for Governing Equation (Matrix Form)

\[
S_{ij} \frac{dU_i}{dt} + (K_{ij} + BK_{ij}CBC)U_j + BK_{ij}BCU_j = 0
\]

\[
S_{ij} = \int_{\Omega} (B_i + w \Delta x W_x \frac{\partial B_i}{\partial x} + w \Delta y \frac{\partial B_i}{\partial y}) B_j d\Omega
\]

\[
K_{ij} = \int_{\Omega} - \frac{\partial B_i}{\partial x} M_x B_i - \frac{\partial B_i}{\partial y} B_j + B_i DB_j + w(\Delta x W_x \frac{\partial B_i}{\partial x} + \Delta y W_y \frac{\partial B_i}{\partial y})(A \frac{\partial B_i}{\partial x} + B \frac{\partial B_i}{\partial y} + NB_j) d\Omega
\]
Computational Flowchart

**INPUT**
- Number of Node, Element and Timestep, Implicitness, Timestep size, Roughness Coeff.

**Initial Conditions for Time Marching Loop**

**Construct Upwinding Matrix**

**Construct Flow Matrix**

**Compute Jacobians for Each Element**

**Build Global Matrix**

**Introduce U/S and D/S Boundary Condition**

**Solve Resulting Equation by Newton-Raphson Method**

**Check the Convergence and Update** $U^{n+1}$ by

$$U^{n+1} = U^n + \Delta U^n$$

**OUTPUT**
- Velocity Vector and Flow Depth
- Switch $h$, $u$, $v$ for Next Timestep

**Calibrate and Verify the Simulation Results**

**SU/PG Scheme for Free Surface Flow**

Each timestep loop
Dry/Wet Problems in Natural River

Floodplain

Wet and Dry Area

Island
Deforming Grid Method

- Prediction of stage at \((n+1)\) time by considering the rising rate of water level
- Decision of Dry/Wet for all nodes
- Elimination of drying nodes and elements
- Performance of renumbering to minimize bandwidth
- Realistic representation of Dry/Wet condition
- Reduction of computation time by eliminating drying nodes and elements
Deforming Grid Method

- Prediction of Stage at (n+1) Time
- Decision of Dry/Wet for All Nodes
- Elimination of Drying Nodes and Elements
- Renumbering
- Simulation
Deforming Grid Method

- Renumbering to minimize bandwidth
- Specification of new boundary conditions

Before Renumbering

After Renumbering
Transition Element Method

- Method to analyze transition region which exist under dry/wet condition
- Improvement of computation for boundary of wetting and drying area
- Effective computation for flooding area

Concept of Domain Coefficient, $\sigma$

Vertical section through a control volume of the flow domain
Hybrid method (FE and FV method)

Numerical problems which produce under Dry Bed
- Difficulty of numerical computation for flow interchange between dry and wet bed

![Diagram showing Riemann problem in dry bed and solutions with dry bed](image)

Deep water \((h_L, U_L)\)

Dry Bed \((h_R, U_R) = (0, 0)\)

Dam or Gate

Wet bed  Dry bed

\[ S_L = u_L - a_L \]

\[ S_L = u_L + 2a_L \]

Riemann problem in dry bed  Solutions w/ dry bed
Hybrid method

Fictitious Mesh in Computation Area

Mesh Structure
Hybrid method

Computation Flowchart

Input
Cross section Data, Initial Condition, Boundary Condition, Output Time, Manning’s Roughness Factor

Create Element
Computation of vertex coordinates - Computation of Element Area, Computation of Angle between Element and Boundary

Each time step
CFL Condition (explicit method)
Computation of Time Interval \( \Delta T \)

HLL Riemann Solver

4th RUNGE-KUTTA

TIME = Output time

Output
Central Coordinates of element, Depth, Flow velocity

SWEEP

Effect of Source Terms for Computational Direction \( \Rightarrow \) Computation of \( \delta \)

Reconstitution of Flow Data for Each Element

Computation of Value for Fictitious Element of Boundary Condition

From HLL Riemann approximate solution, Computation of Flow Rate in Computational Direction

Computation of WAF Flow Rate Using TVD Method

Update of Value to Next Time

Computation of flow variables by using HLL Riemann approximation solution
Application to U-shaped Channel
Station 4, Outer

Station 4, Inner

(a) Station 4 (outer side)  
(b) Station 4 (inner side)
(a) Station 6 (outer side)

(b) Station 6 (inner side)
Velocity Distributions

(a) $t = 7.56s$

(b) $t = 10.08s$
Simple Channel with Dry Bed

- Size of simple channel
  - Length: 50m
  - Bottom slope: 0.1
  - Number of node: 310, Number of element: 270

- Computation Time
  - Time step: 0.01hr
  - Total computation time: 1hr
Bottom elevation

Velocity distribution
Applications to Simple Channel

- Size of simple channel
  - Length: 50m
  - Bottom slope: 0.1
  - Number of node: 310, Number of element: 270

- Computation Time
  - Time step: 0.01hr
  - Total computational time: 1hr
Bottom elevation

U/S Boundary Condition

D/S Boundary Condition
Trapezoidal Channels (partly dry side slope)

Size of simple channel
- Length: 300m, Width: 40m
- Channel with trapezoidal cross section

Boundary Condition
- U/S Boundary condition: 80m³/s
- D/S Boundary condition: 4.0m
Construction of finite element mesh

Velocity distribution

Depth contour
Mesh Resolution Effects with Dry Bed

Bottom Elevation = 0.0 m

H = 8.0 m

80 m³/s

coarse element

refined element

Channel
Application to Milyang-river

Milyang

Main stream of Nakdong

Subjective Element of Wet/Dry Area
Velocity distribution

Water surface elevation
Application to Nakdong-river
Application to Keum River

Kum-river

Section of Wet/Dry Area
Upstream of Kum River
Velocity distribution and Water surface elevation
Conclusions(1)

(1) The challenging problem facing two-dimensional model is the treatment of wet and dry areas. This situation is encountered in most practical river and coastal engineering problems.

(2) To solve the dry/wet problems, deforming grid method, transition element method, and hybrid method are adopted.

(3) The model is verified by applying to U-shaped laboratory channel. The simulation result agree with observed data for various flow conditions.
Conclusions(2)

(4) Trapezoidal channel with partly dry side slopes, straight channel with various drying conditions, are examined for flow model validations.

(5) RAM2 model shows reasonable flow distribution compared with existing model in dry area simulation in Milyang-river, Nakdong-river and Keum-river.

(6) RAM2 model can be used as a hydrodynamic input for water quality and sediment transport model as well as be used for analyzing dam-break flow, levee-break flow and rapidly varied flow.
Thank You ..