



Sustainable Water Resources Research Center



Modeling of 2D Flood Flow Analysis by Finite Element Method

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Kun-Yeun Han, Sang-Ho Kim, Seung-Yong Choi

Kyungpook National Univ., Korea

Outline

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5

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Introductions

- 2D hydrodynamic model can provide good estimates of complex features in the flow around islands and obstructions, flow at confluence and flow in braided channel.
- The challenging problem facing two-dimensional FE model is the treatment of wet and dry areas. This situation is encountered in most practical river such as flood propagation, dam break analysis, tidal processes and so on.

Objectives of the Study

- To develop an accurate and robust two-dimensional finite element method with wet and dry simulation in complex natural rivers.

Scope of the Study

Development of Dry/Wet Algorithm

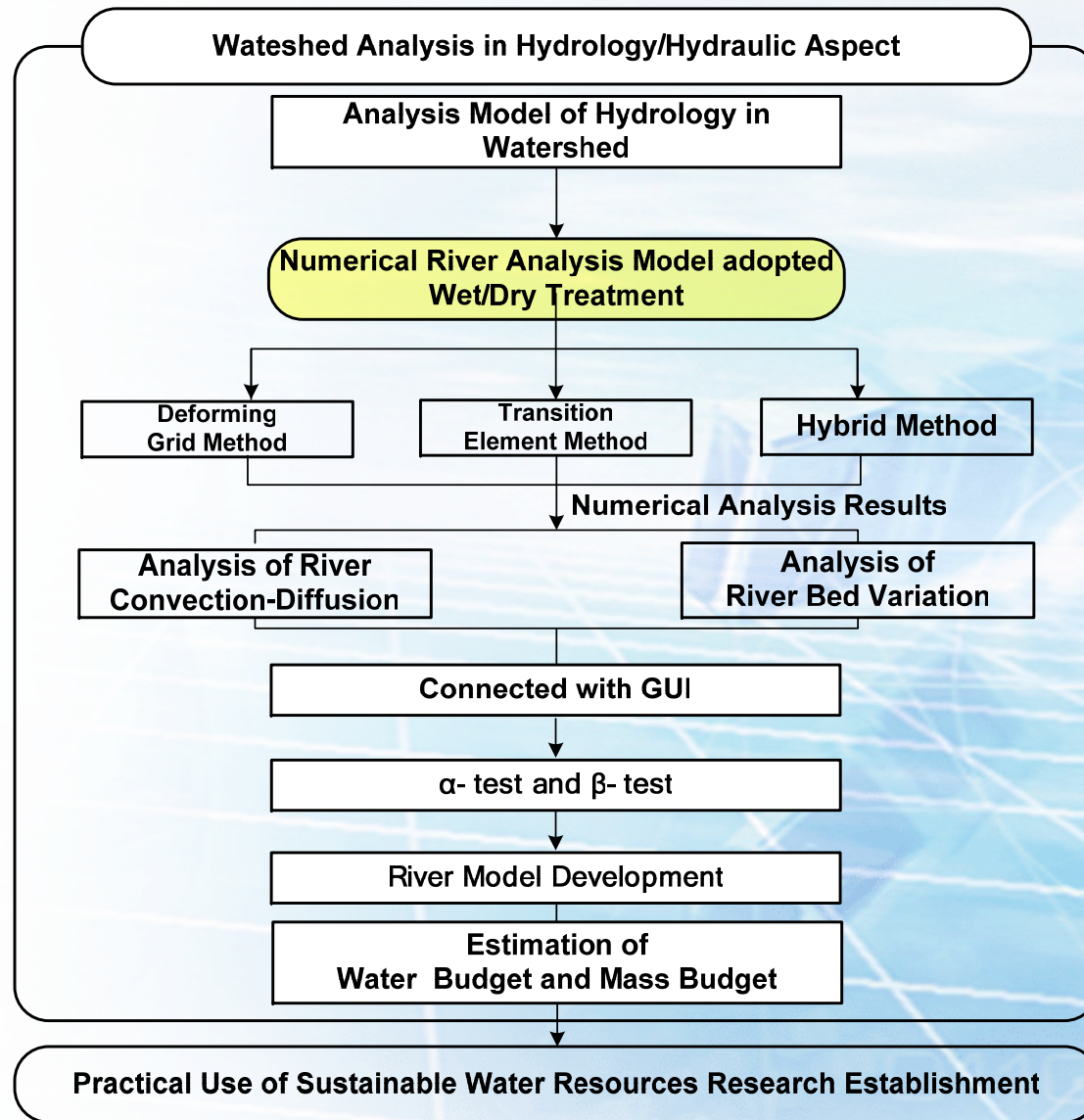
- Development of Deforming Grid Method
- Development of Transition Element Method
- Development of Hybrid Method

Application of Dry/Wet Algorithm

- Laboratory U-shaped Channels
- 2D Dambreak-wave, Several cases with dry state

Application of RAM2 Model to natural channel

- Application of RAM2 Model for Milyang-river, Nakdong-river, Kum-river



Governing Equation (Matrix Form)

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} + D = 0$$

$$U = (h, p, q)^T$$

$$F = \begin{pmatrix} p \\ \frac{p^2}{h} + \frac{gh^2}{2} \\ \frac{pq}{h} \end{pmatrix} = M_x U = \begin{bmatrix} 0 & 1 & 0 \\ \frac{gh}{2} & u & 0 \\ 0 & 0 & u \end{bmatrix} U \quad G = \begin{pmatrix} q \\ \frac{pq}{h} \\ \frac{q^2}{h} + \frac{gh^2}{2} \end{pmatrix} = M_y U = \begin{bmatrix} 0 & 0 & 1 \\ 0 & v & 0 \\ \frac{gh}{2} & 0 & v \end{bmatrix} U$$

$$D = \begin{pmatrix} gh \frac{\partial z_0}{\partial x} + gn^2 \frac{p(p^2 + q^2)^{1/2}}{h^{7/3}} - \frac{\partial}{\partial x} \left(\varepsilon_{xx} \frac{\partial p}{\partial x} \right) - \frac{\partial}{\partial y} \left(\varepsilon_{xy} \frac{\partial p}{\partial y} \right) - \frac{p}{h} \left(\frac{i}{2} \right) \\ gh \frac{\partial z_0}{\partial y} + gn^2 \frac{q(p^2 + q^2)^{1/2}}{h^{7/3}} - \frac{\partial}{\partial x} \left(\varepsilon_{yx} \frac{\partial q}{\partial x} \right) - \frac{\partial}{\partial y} \left(\varepsilon_{yy} \frac{\partial q}{\partial y} \right) - \frac{q}{h} \left(\frac{i}{2} \right) \end{pmatrix}$$

Weighting Functions

$$\int_{\Omega} B_i^T \left(\frac{\partial \hat{U}}{\partial \tau} - \frac{\partial x}{\partial \tau} \frac{\partial \hat{U}}{\partial x} - \frac{\partial y}{\partial \tau} \frac{\partial \hat{U}}{\partial y} + A \frac{\partial \hat{U}}{\partial x} + B \frac{\partial \hat{U}}{\partial y} + D \right) d\Omega = 0$$

Weighting Functions

Bubnov-Galerkin Scheme

$$[B'_i] = [B_i]$$

Petrov-Galerkin Scheme

$$[B'_i] = [B_i] + \varepsilon_x \left[A^T \frac{dB_i}{dx} \right] + \varepsilon_y \left[B^T \frac{dB_i}{dx} \right]$$

$$\varepsilon_x = \frac{\Delta x}{|U + c| \sqrt{15}}$$

$$\varepsilon_y = \frac{\Delta y}{|V + c| \sqrt{15}}$$

$$\varepsilon = \frac{2}{\sqrt{15}} \approx 0.50$$

SU/PG Scheme

$$[B'_i] = [B_i] + \alpha \frac{\Delta x}{2} \left[\frac{[A]}{|[A]|} \right]^T \frac{dB_i}{dx} + \alpha \frac{\Delta y}{2} \left[\frac{[B]}{|[B]|} \right]^T \frac{dB_i}{dx}$$

$$[W_x] = [M_x] \begin{bmatrix} \lambda_i \\ |\lambda_i| \end{bmatrix} [M_x]^{-1}$$

$$[W_y] = [M_y] \begin{bmatrix} \lambda_i \\ |\lambda_i| \end{bmatrix} [M_y]^{-1}$$

Upwinding Matrix

Upwinding Matrix of PG Scheme(2D)

$$W_x = \begin{bmatrix} 0 & \frac{1}{|U+c|} & 0 \\ \frac{c^2 - U^2}{|U+c|} & \frac{2U}{|U+c|} & 0 \\ \frac{-UV}{|U+c|} & \frac{V}{|U+c|} & \frac{U}{|U+c|} \end{bmatrix} \quad W_y = \begin{bmatrix} 0 & 0 & \frac{1}{|V+c|} \\ \frac{-UV}{|V+c|} & \frac{V}{|V+c|} & \frac{U}{|V+c|} \\ \frac{c^2 - V^2}{|V+c|} & 0 & \frac{2V}{|V+c|} \end{bmatrix}$$

Upwinding Matrix of SU/PG(2D)

$$[W_x] = -\frac{1}{2c} \begin{bmatrix} 1 & 0 & 1 \\ U-c & 0 & U+c \\ V & 1 & V \end{bmatrix} \begin{bmatrix} \frac{U-c}{|U-c|} & 0 & 0 \\ 0 & \frac{1}{2} \left(\frac{U+c}{|U+c|} + \frac{U-c}{|U-c|} \right) & 0 \\ 0 & 0 & \frac{U+c}{|U+c|} \end{bmatrix} \begin{bmatrix} -(U+c) & 1 & 0 \\ 2cV & 0 & -2c \\ U-c & -1 & 0 \end{bmatrix}$$

$$[W_y] = \frac{1}{2c} \begin{bmatrix} 1 & 0 & 1 \\ U & 1 & U \\ V-c & 1 & V+c \end{bmatrix} \begin{bmatrix} \frac{V-c}{|V-c|} & 0 & 0 \\ 0 & \frac{1}{2} \left(\frac{V-c}{|V-c|} + \frac{V+c}{|V+c|} \right) & 0 \\ 0 & 0 & \frac{V+c}{|V+c|} \end{bmatrix} \begin{bmatrix} (V+c) & 0 & -1 \\ -2cU & 2c & 0 \\ -(V-c) & 0 & 1 \end{bmatrix}$$

Implementation of SU/PG Method

Implementation of SU/PG for Governing Equation

$$\begin{bmatrix} B_i & 0 & 0 \\ 0 & B_i & 0 \\ 0 & 0 & B_i \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} + \alpha \Delta x [W_x] \frac{\partial}{\partial x} \begin{bmatrix} B_i & 0 & 0 \\ 0 & B_i & 0 \\ 0 & 0 & B_i \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} + \alpha \Delta y [W_y] \frac{\partial}{\partial y} \begin{bmatrix} B_i & 0 & 0 \\ 0 & B_i & 0 \\ 0 & 0 & B_i \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} = \{0\}$$

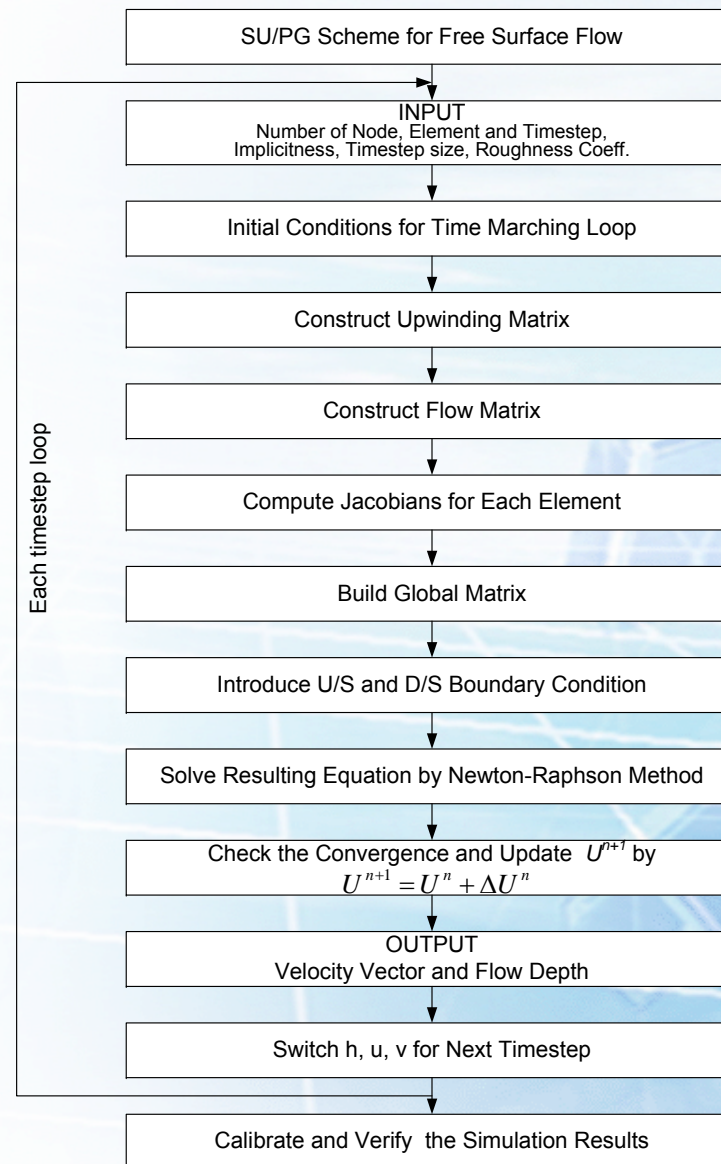
Implementation of SU/PG for Governing Equation (Matrix Form)

$$S_{ij} \frac{dU_i}{dt} + (K_{ij} + \overline{BK}_{ij} \overline{CBC}) U_j + \overline{BK}_{ij} \overline{BC} \hat{U}_j = 0$$

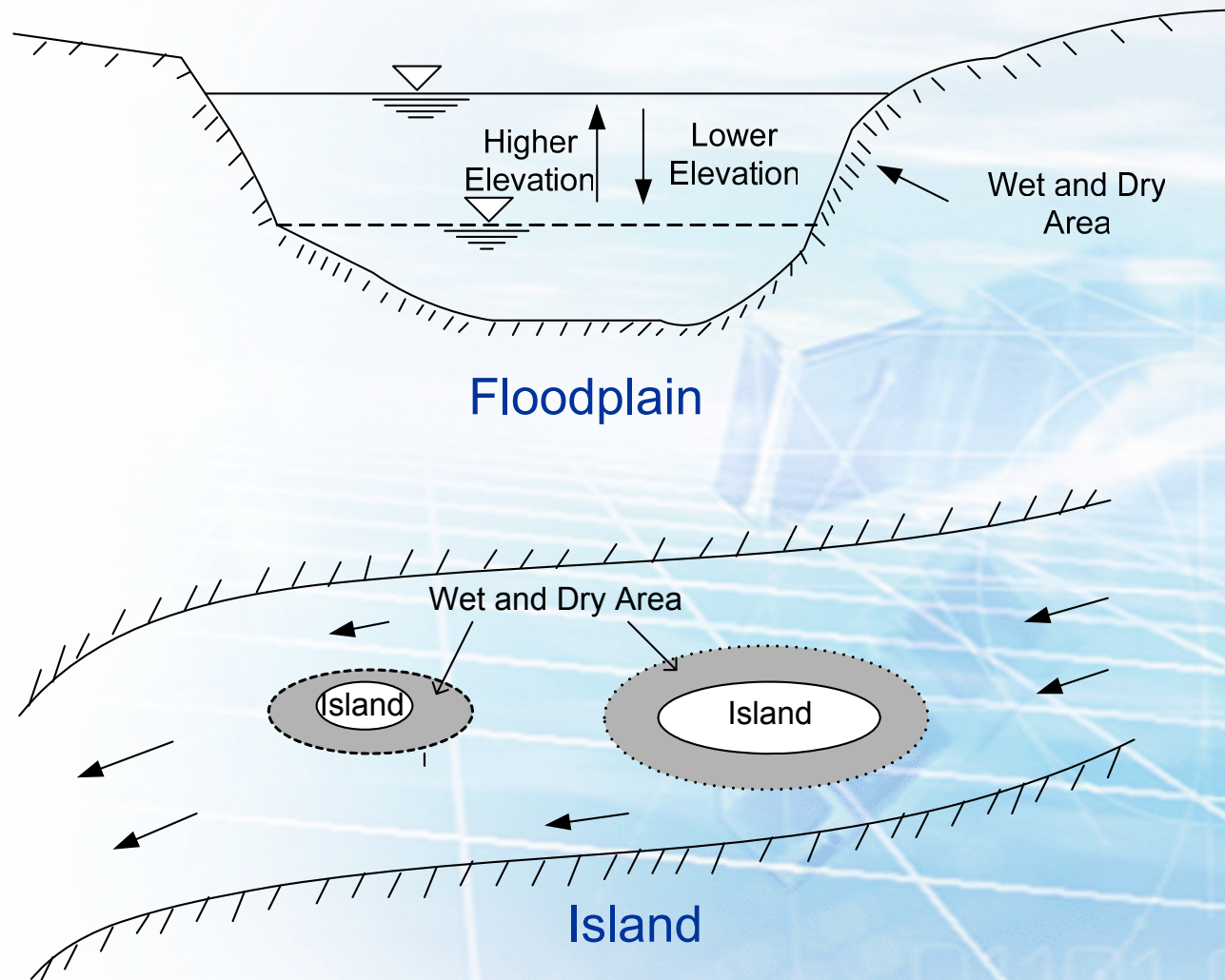
$$S_{ij} = \int_{\Omega} (B_i + w \Delta x W_x \frac{\partial B_i}{\partial x} + w \Delta y \frac{\partial B_i}{\partial y}) B_j d\Omega$$

$$K_{ij} = \int_{\Omega} -\frac{\partial B_i}{\partial x} M_x B_i - \frac{\partial B_i}{\partial y} B_j + B_i D B_j + w (\Delta x W_x \frac{\partial B_i}{\partial x} + \Delta y W_y \frac{\partial B_i}{\partial y}) (A \frac{\partial B_i}{\partial x} + B \frac{\partial B_i}{\partial y} + N B_j) d\Omega$$

Computational Flowchart



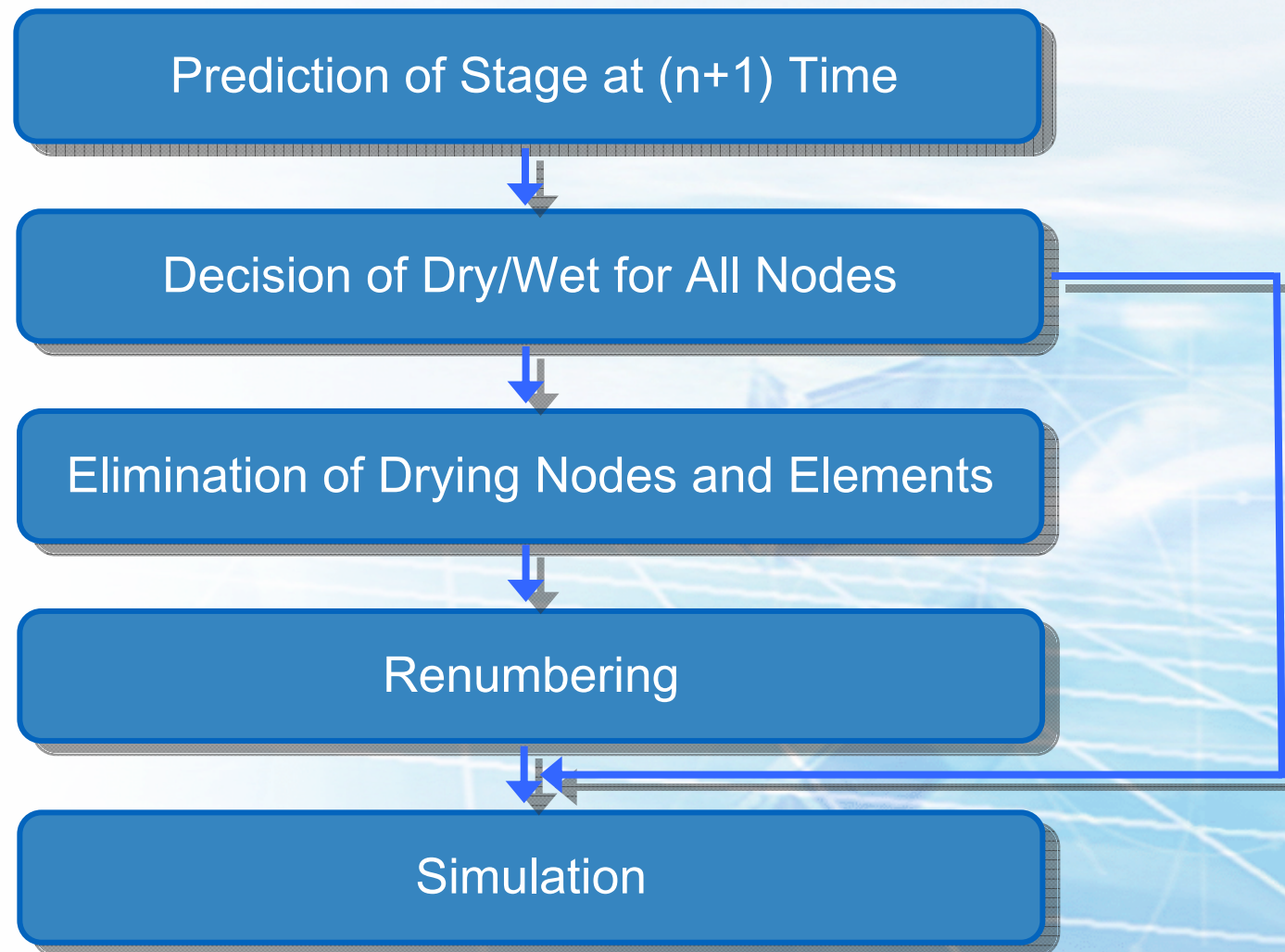
Dry/Wet Problems in Natural River



Deforming Grid Method

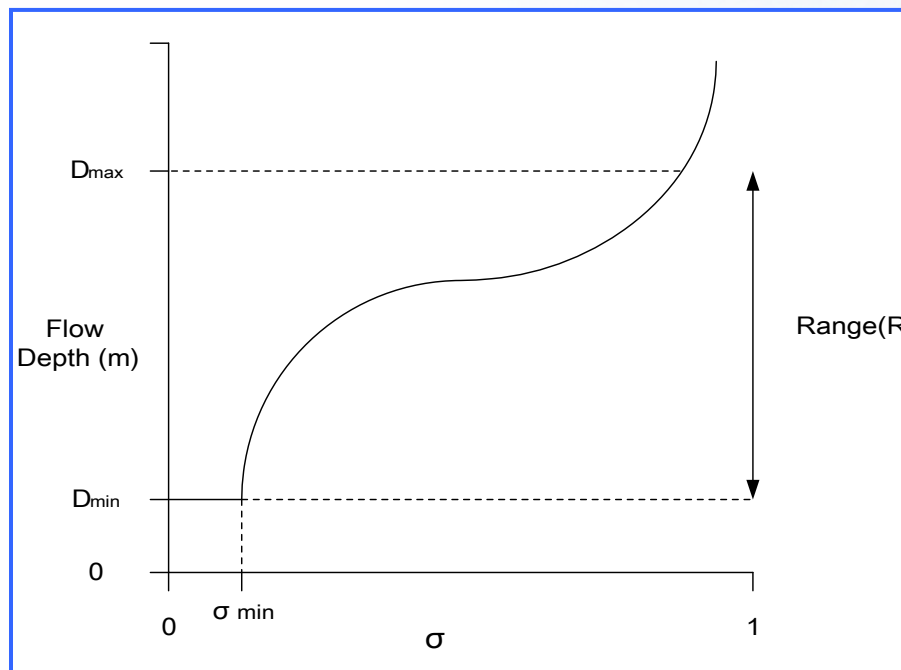
- Prediction of stage at $(n+1)$ time by considering the rising rate of water level
- Decision of Dry/Wet for all nodes
- Elimination of drying nodes and elements
- Performance of renumbering to minimize bandwidth
- Realistic representation of Dry/Wet condition
- Reduction of computation time by eliminating drying nodes and elements

Deforming Grid Method

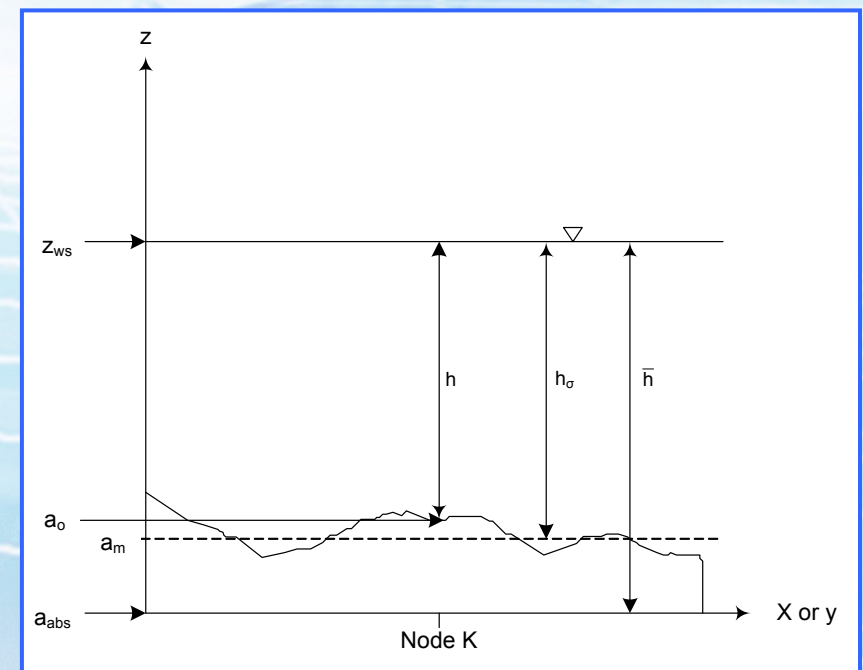


Transition Element Method

- Method to analyze transition region which exist under dry/wet condition
- Improvement of computation for boundary of wetting and drying area
- Effective computation for flooding area



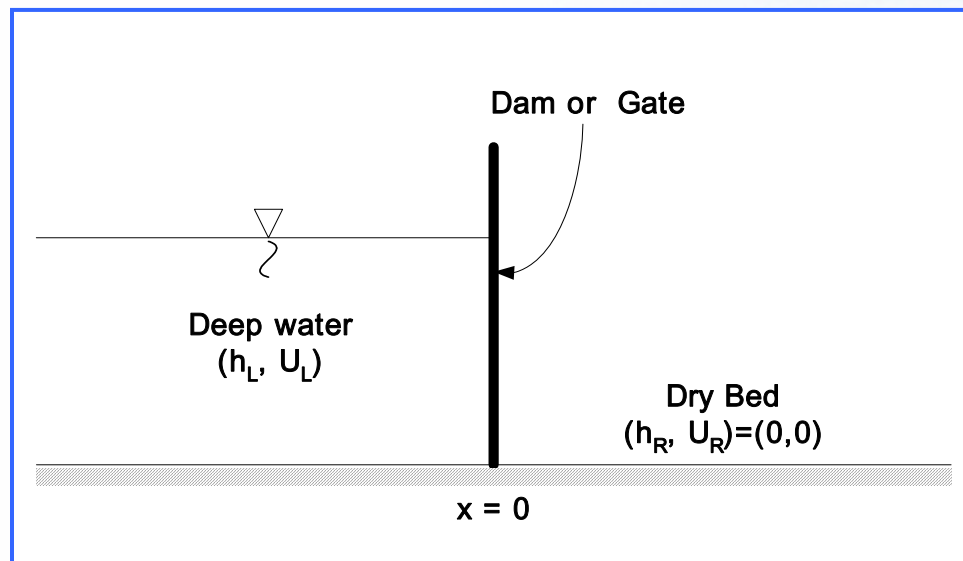
Concept of Domain Coefficient, σ



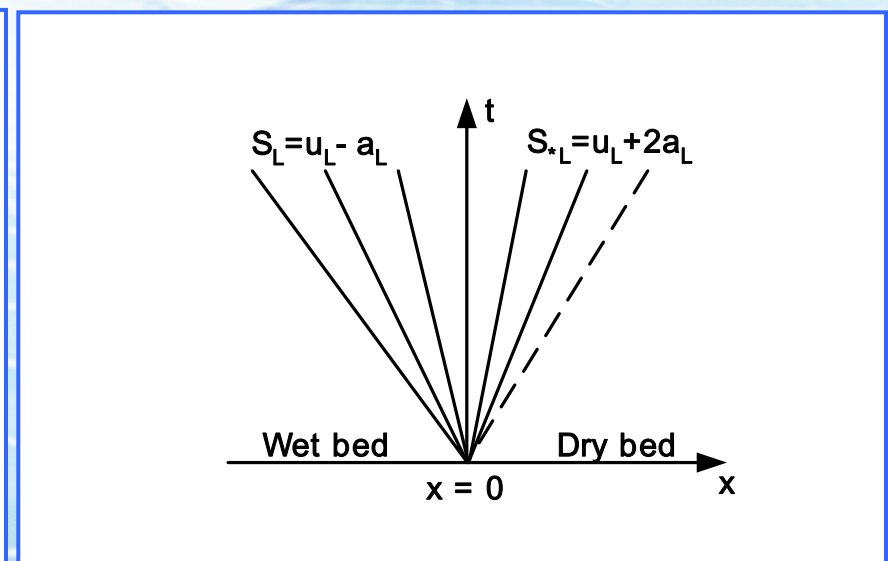
Vertical section through a control volume of the flow domain

Hybrid method (FE and FV method)

- Numerical problems which produce under Dry Bed
 - ◆ Difficulty of numerical computation for flow interchange between dry and wet bed

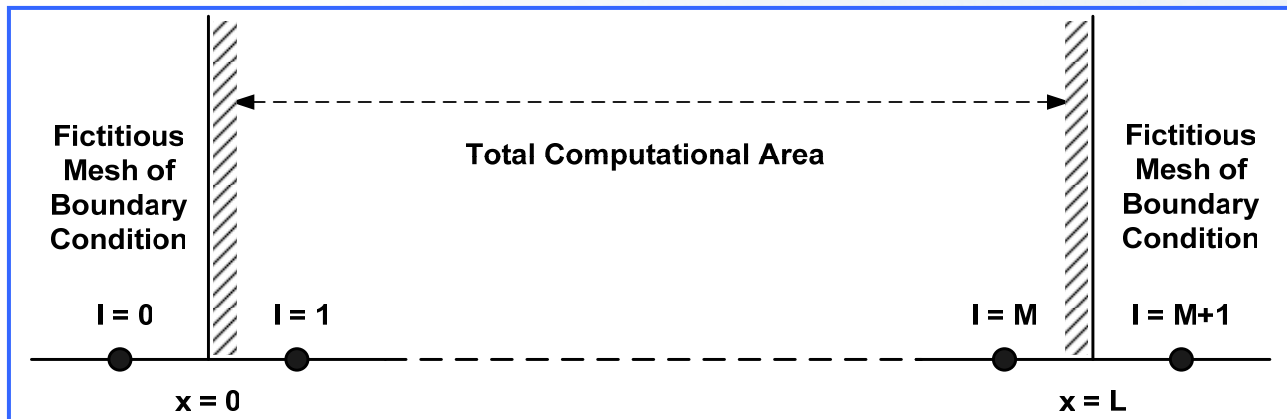


Riemann problem in dry bed

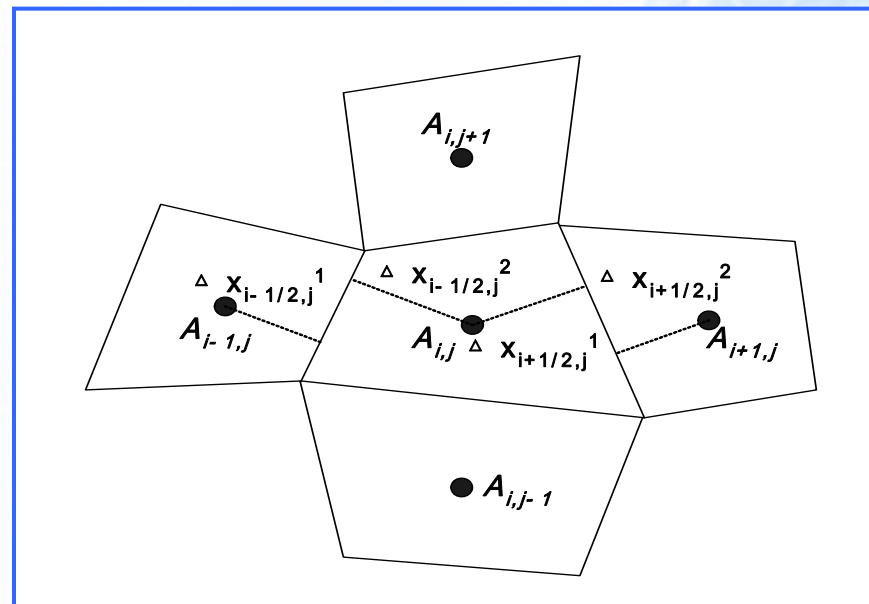


Solutions w/ dry bed

Hybrid method

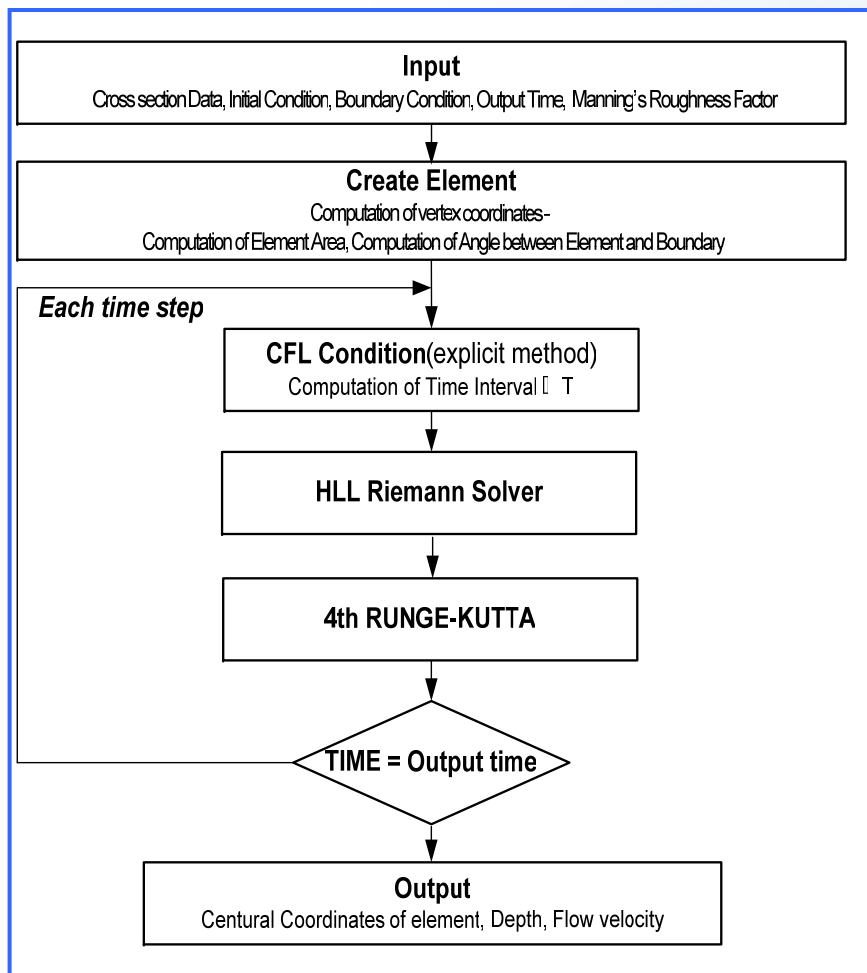


Fictitious Mesh in Computation Area

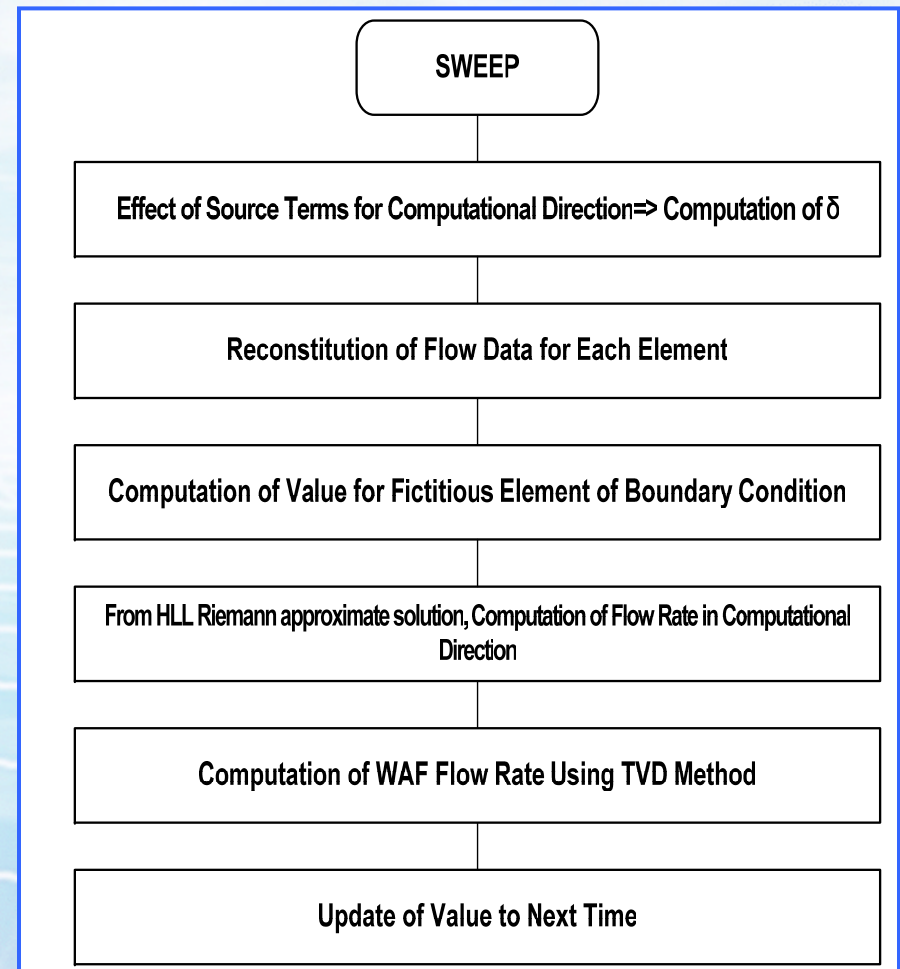


Mesh Structure

Hybrid method

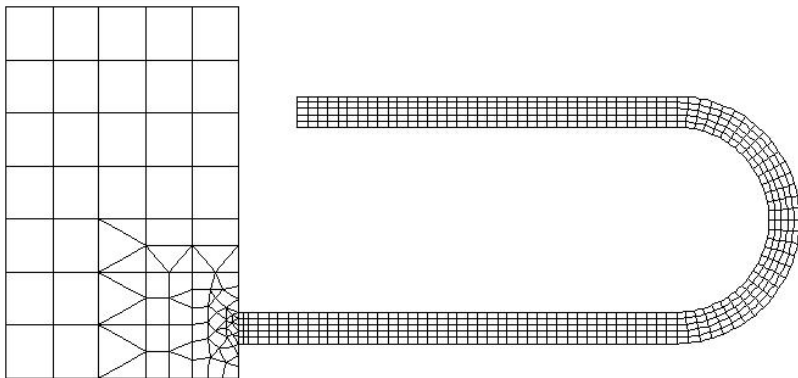
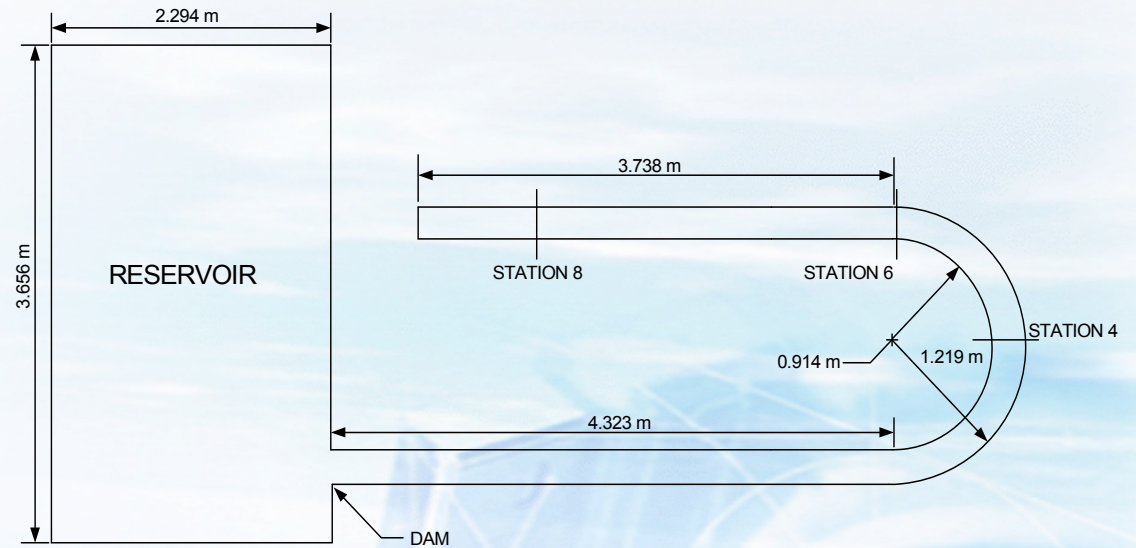


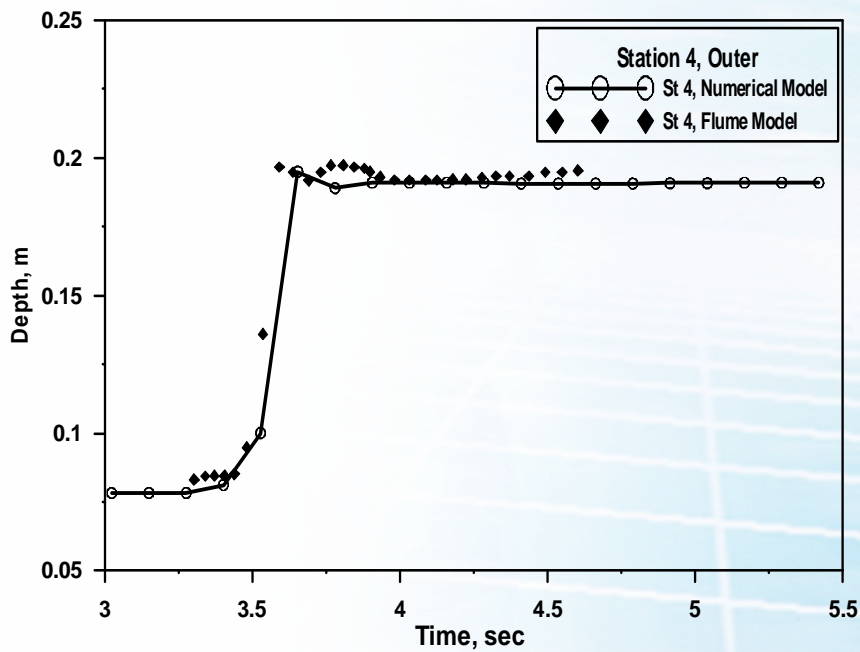
Computation Flowchart



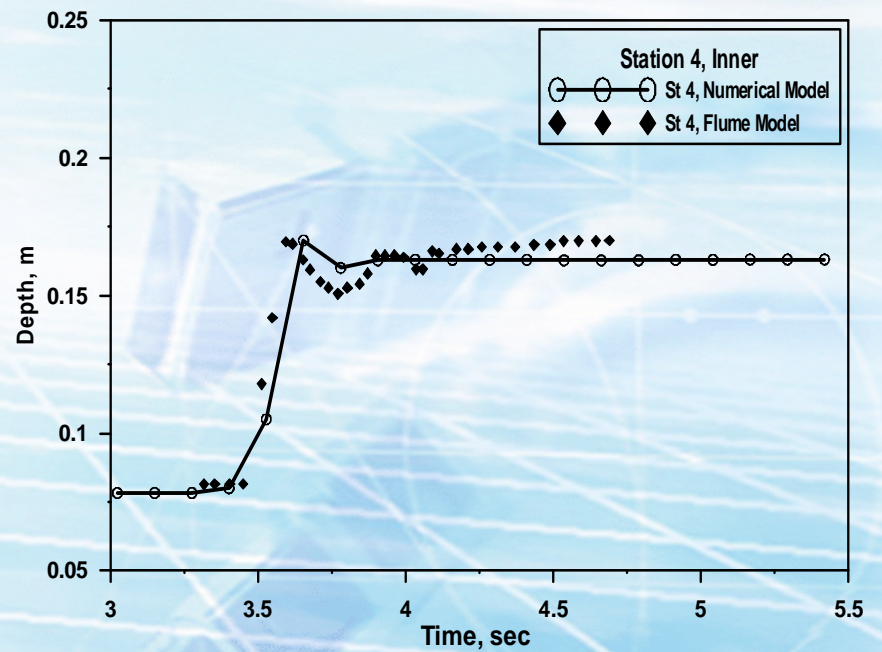
Computation of flow variables by using HLL Riemann approximation solution

Application to U-shaped Channel

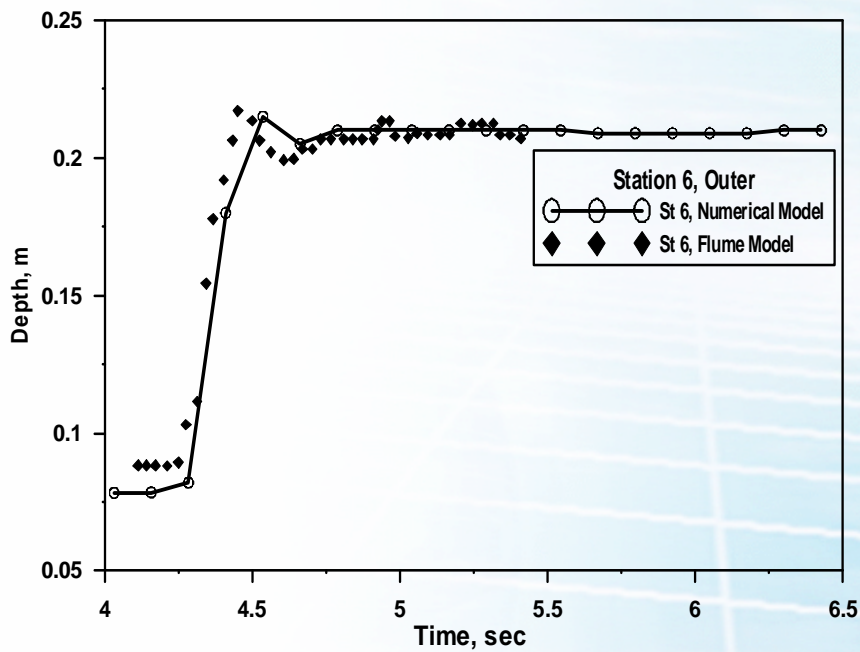




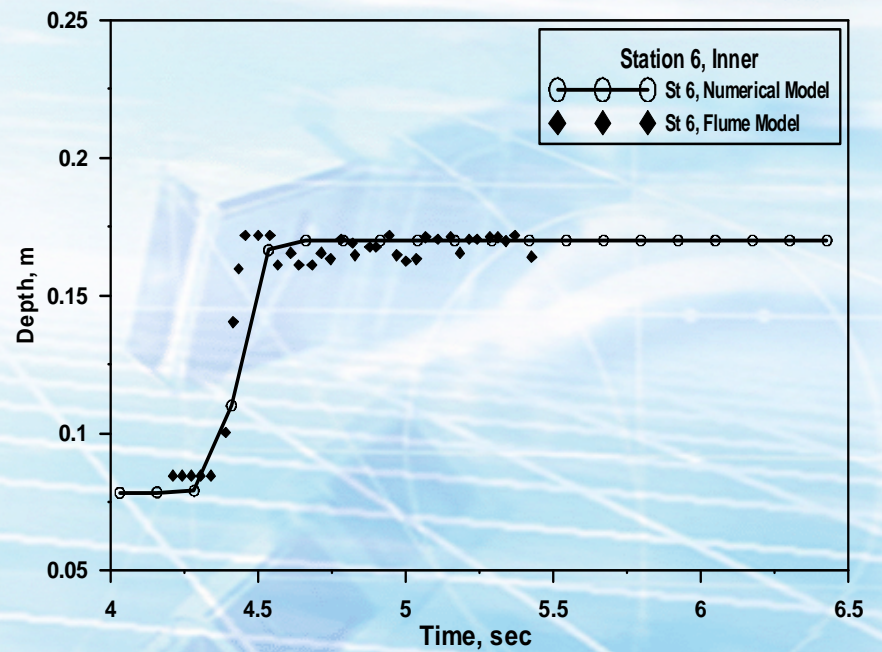
(a) Station 4 (outer side)



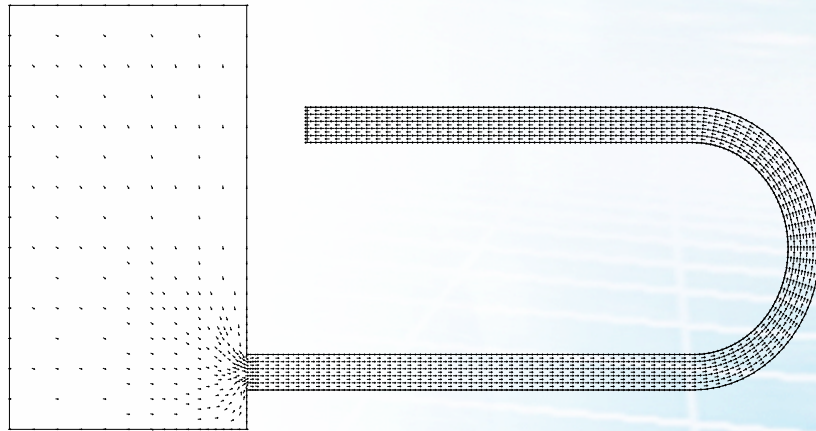
(b) Station 4 (inner side)



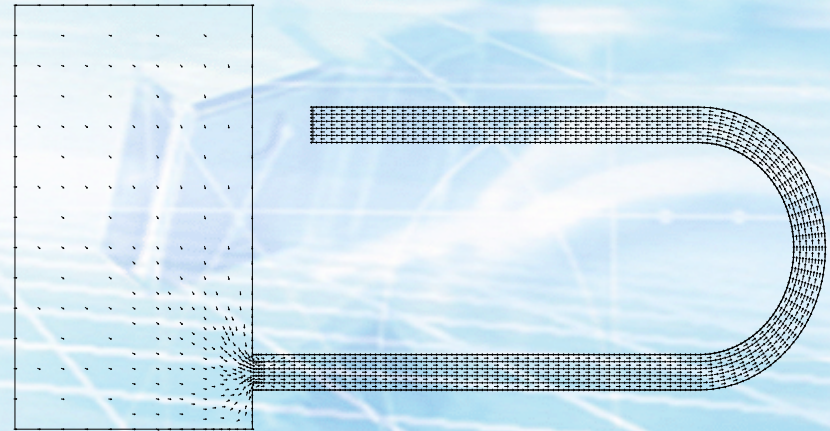
(a) Station 6 (outer side)



(b) Station 6 (inner side)



(a) $t = 7.56\text{s}$

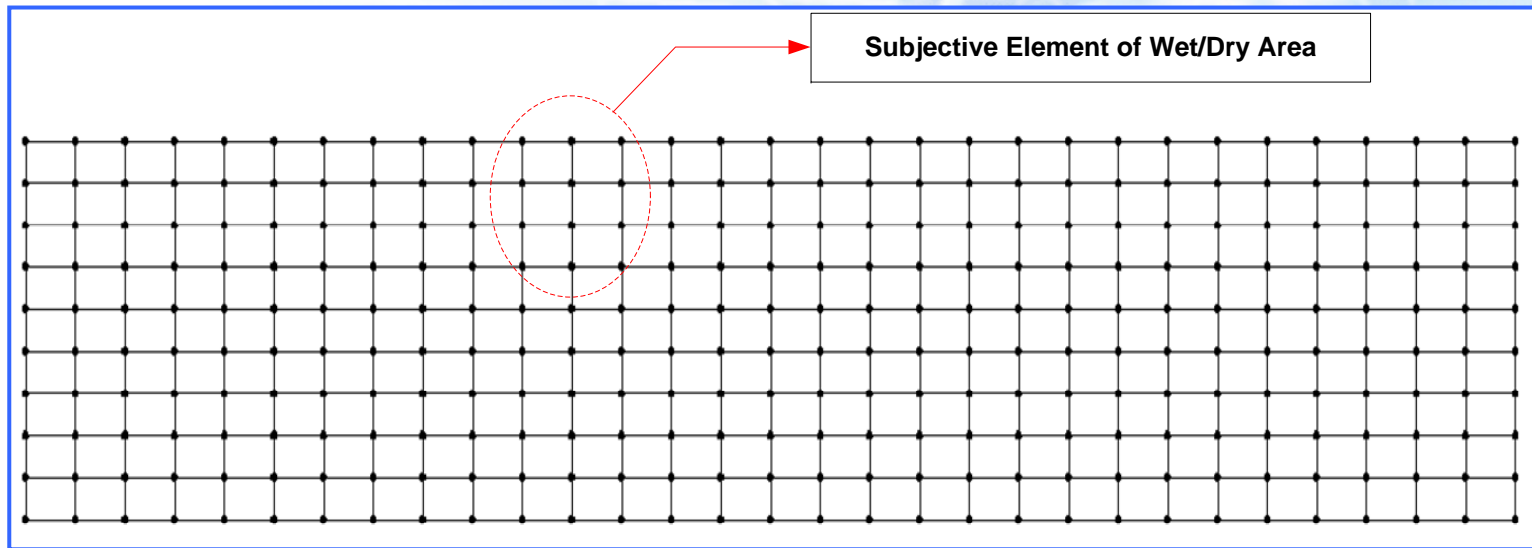


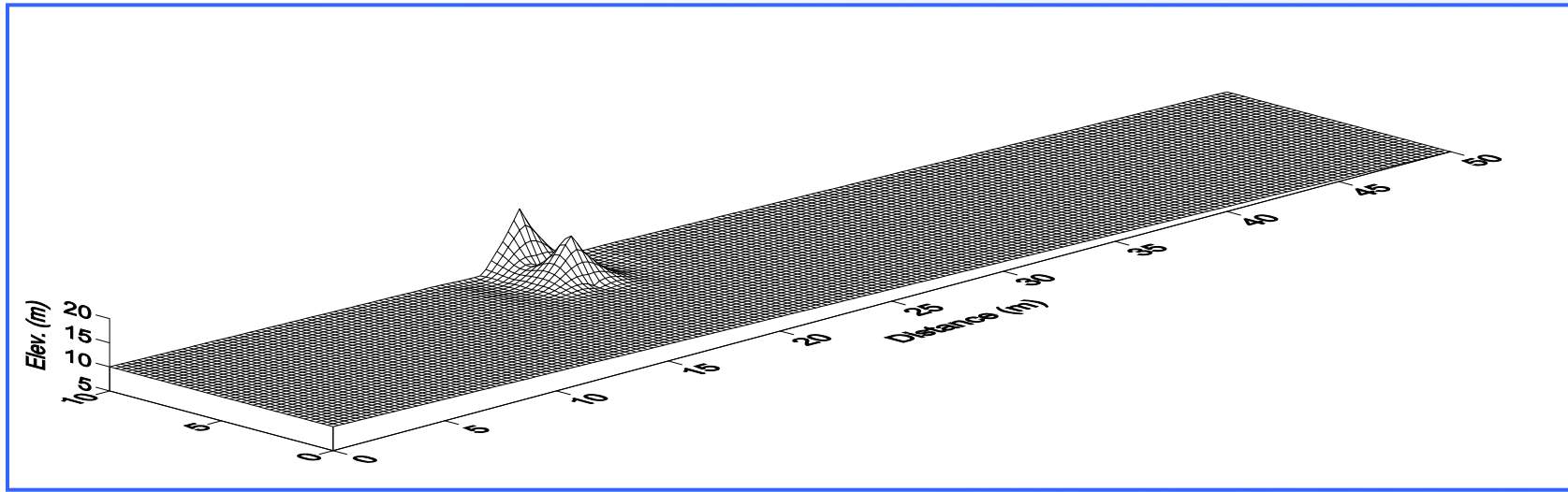
(b) $t = 10.08\text{s}$

Velocity Distributions

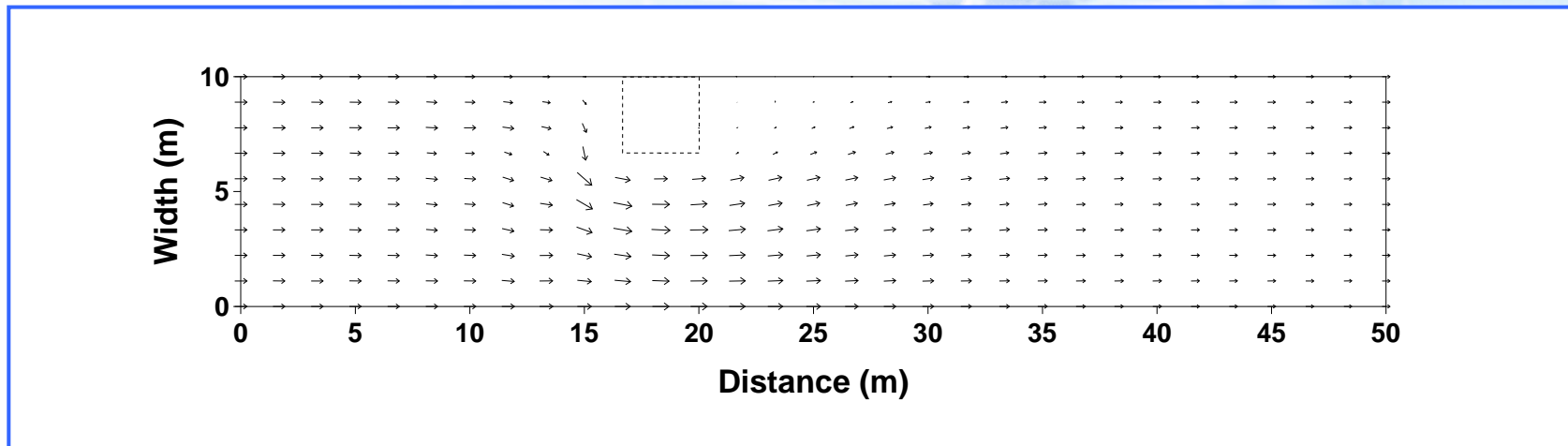
Simple Channel with Dry Bed

- Size of simple channel
 - ◆ Length : 50m
 - ◆ Bottom slope : 0.1
 - ◆ Number of node : 310, Number of element : 270
- Computation Time
 - ◆ Time step : 0.01hr
 - ◆ Total computation time : 1hr





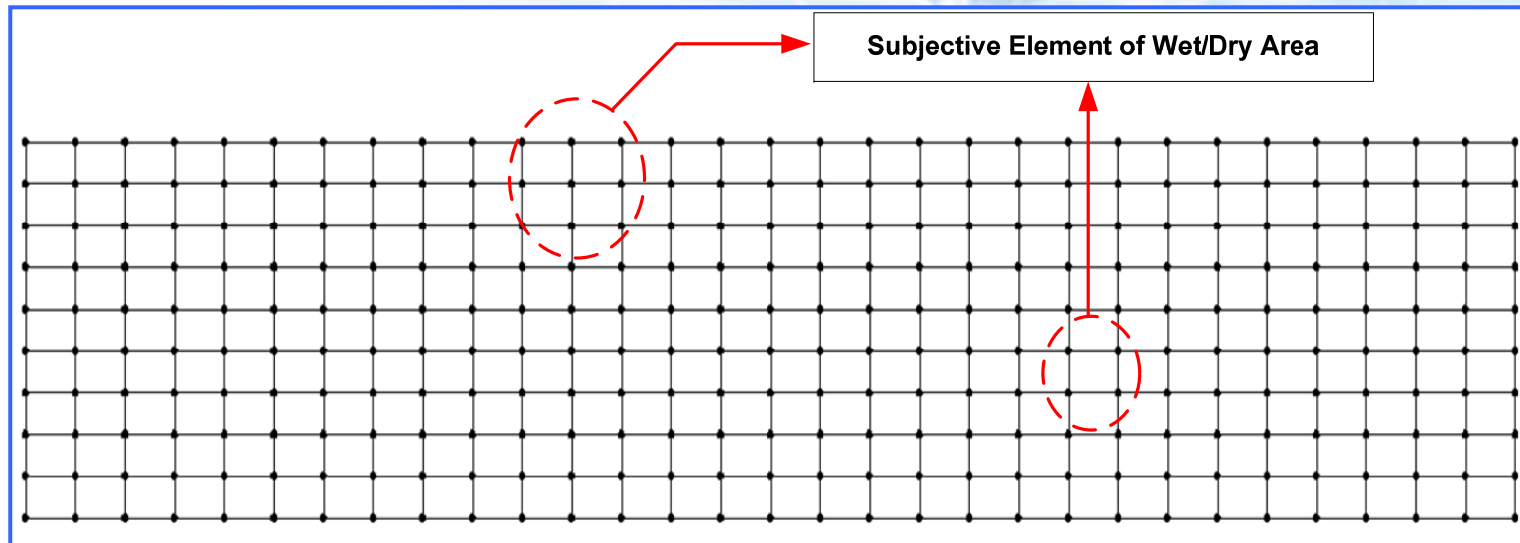
Bottom elevation

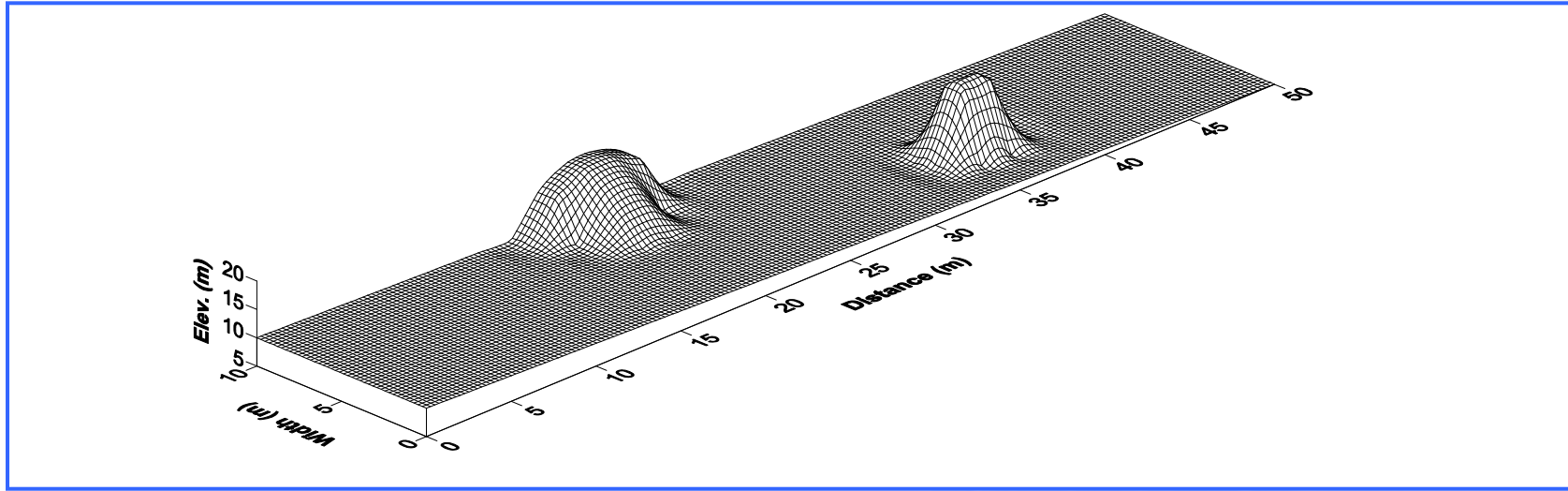


Velocity distribution

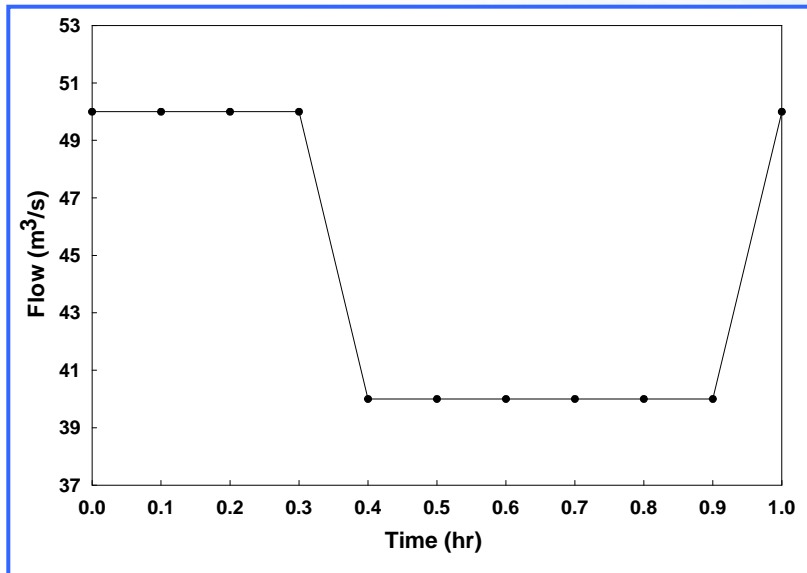
Applications to Simple Channel

- Size of simple channel
 - ◆ Length : 50m
 - ◆ Bottom slope : 0.1
 - ◆ Number of node : 310, Number of element : 270
- Computation Time
 - ◆ Time step : 0.01hr
 - ◆ Total computational time : 1hr

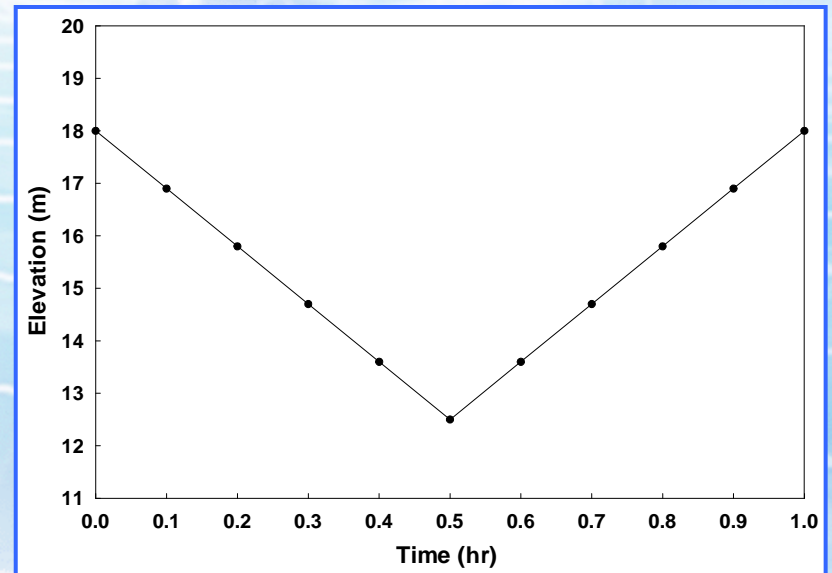




Bottom elevation

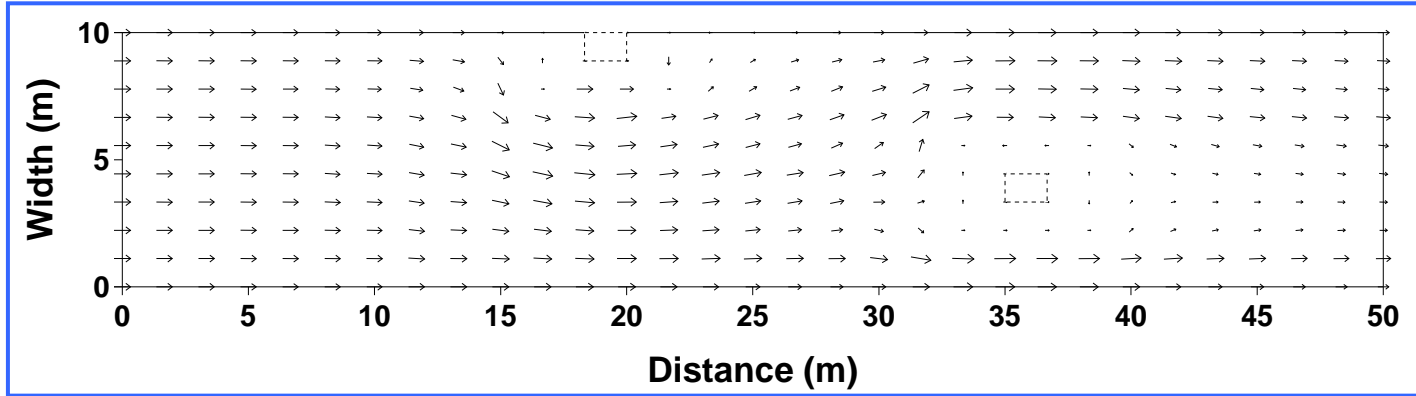


U/S Boundary Condition

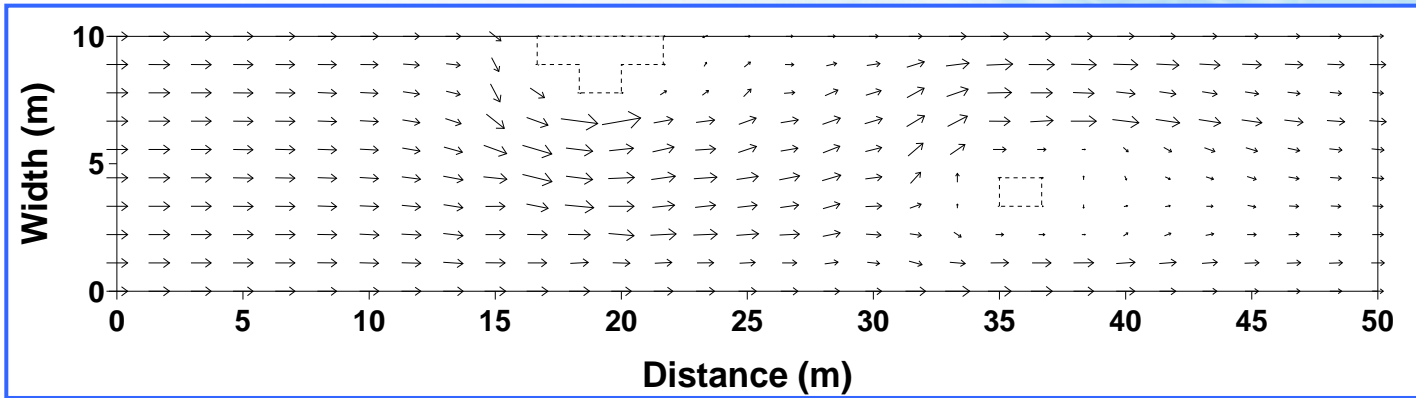


D/S Boundary Condition

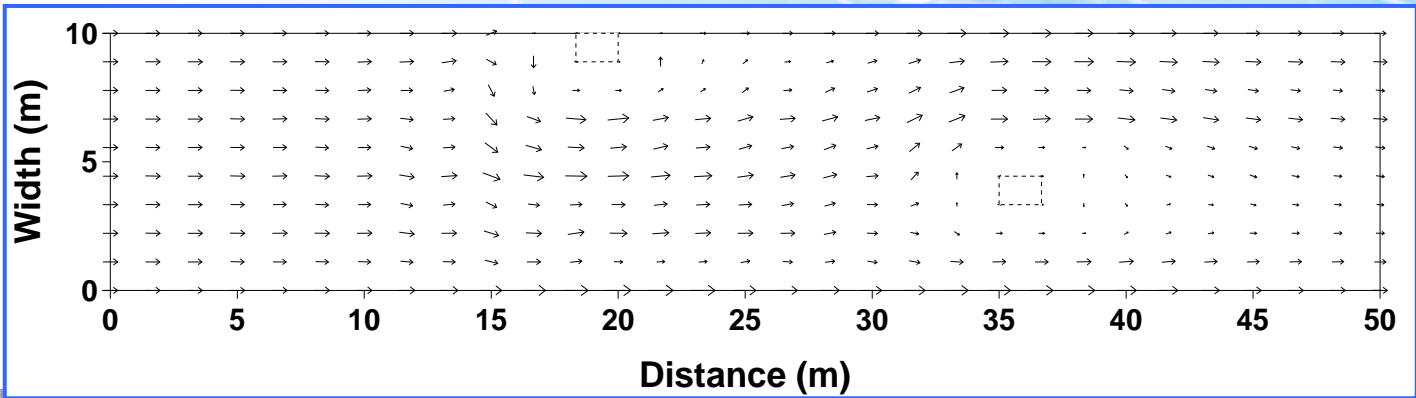
0.2 hr



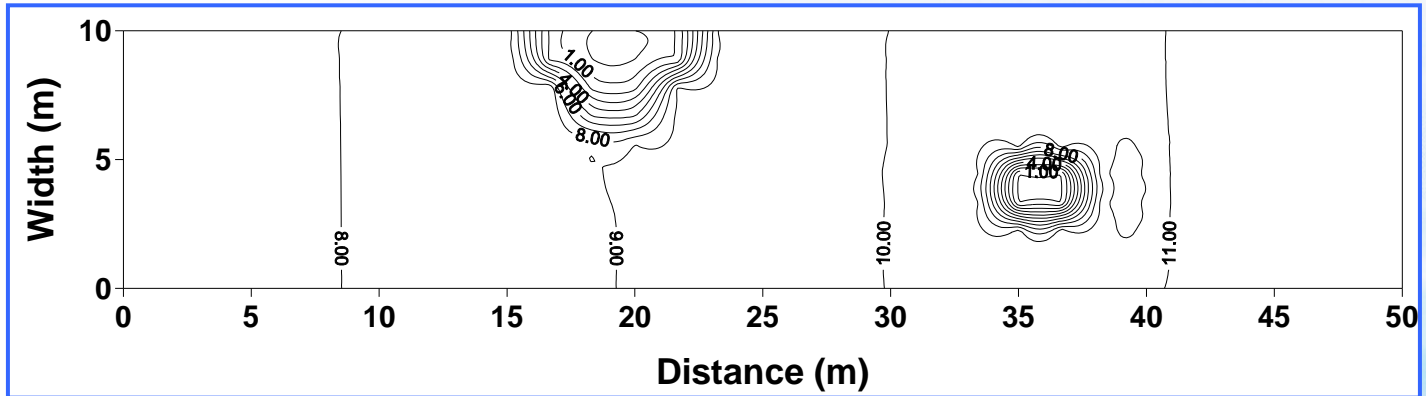
0.4 hr



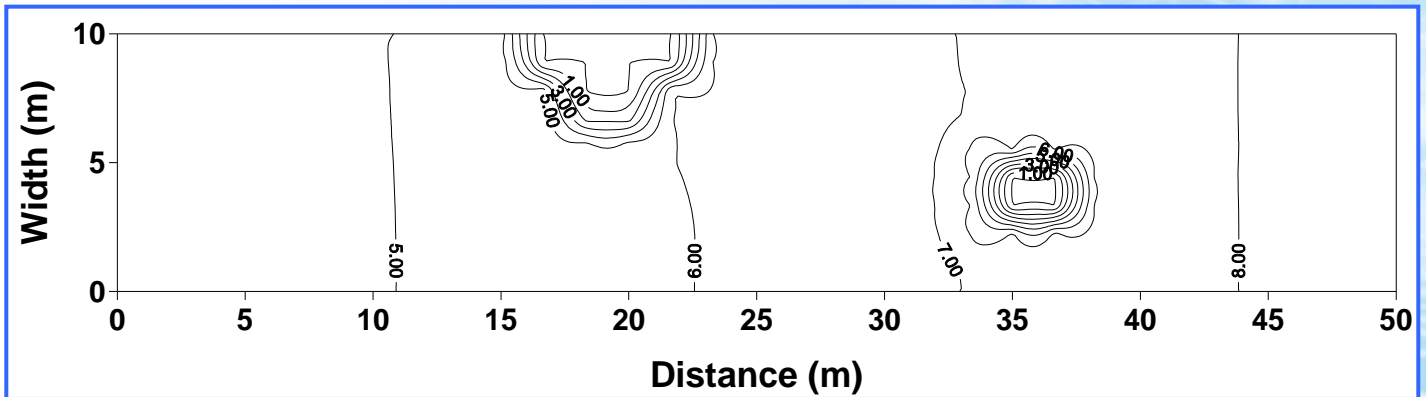
1.0 hr



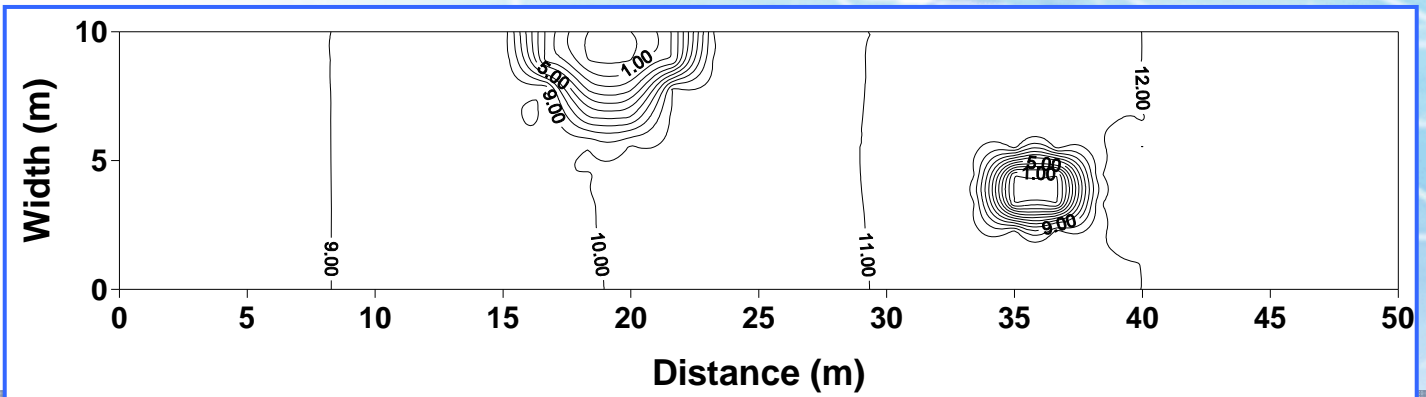
0.2 hr



0.4 hr

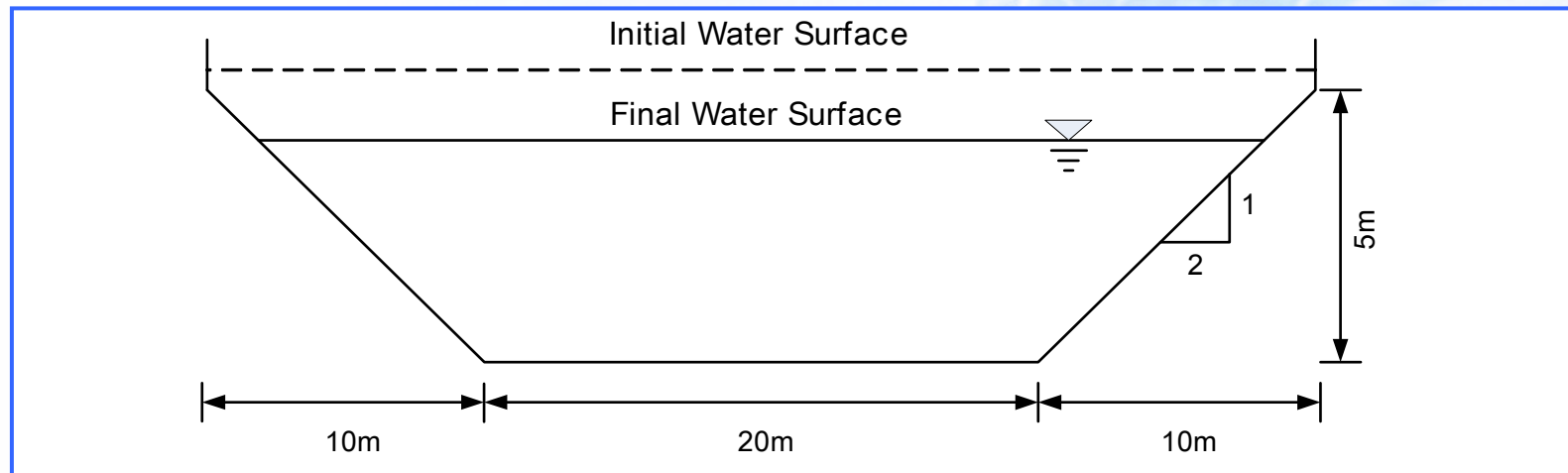


1.0 hr

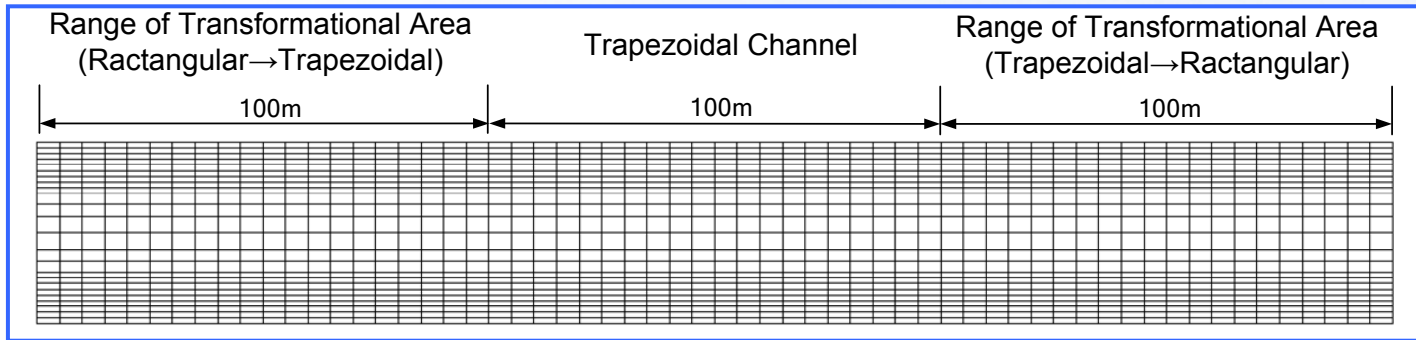


Trapezoidal Channels (partly dry side slope)

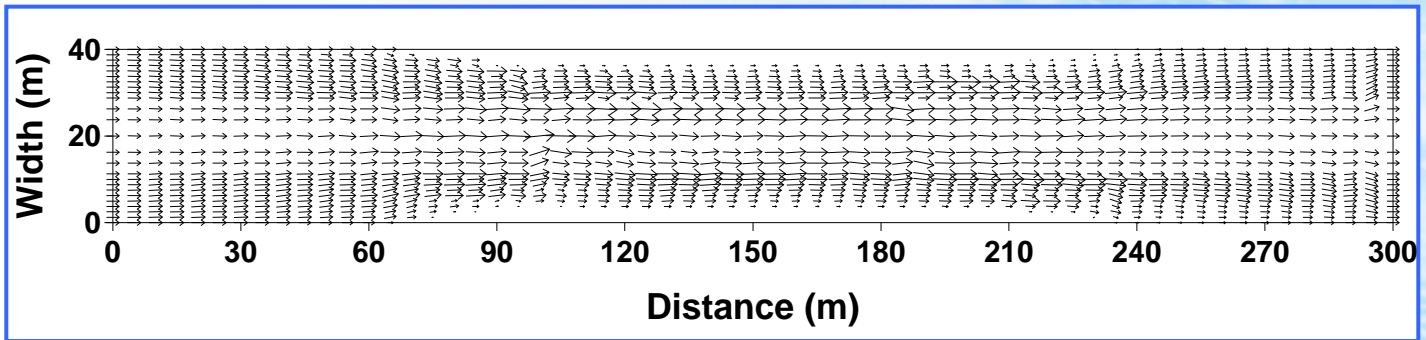
- Size of simple channel
 - ◆ Length : 300m, Width : 40m
 - ◆ Channel with trapezoidal cross section
- Boundary Condition
 - ◆ U/S Boundary condition : $80\text{m}^3/\text{s}$
 - ◆ D/S Boundary condition : 4.0m



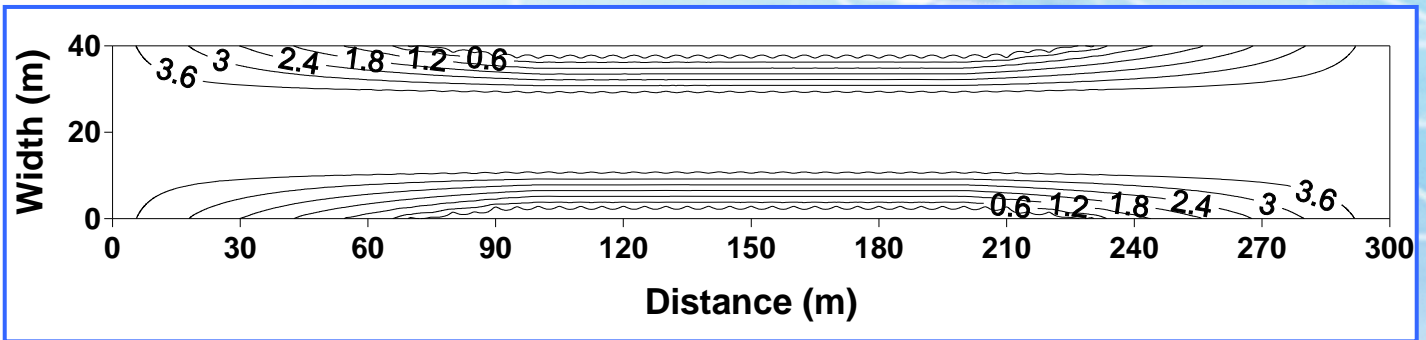
Trapezoidal cross section



Construction of finite element mesh

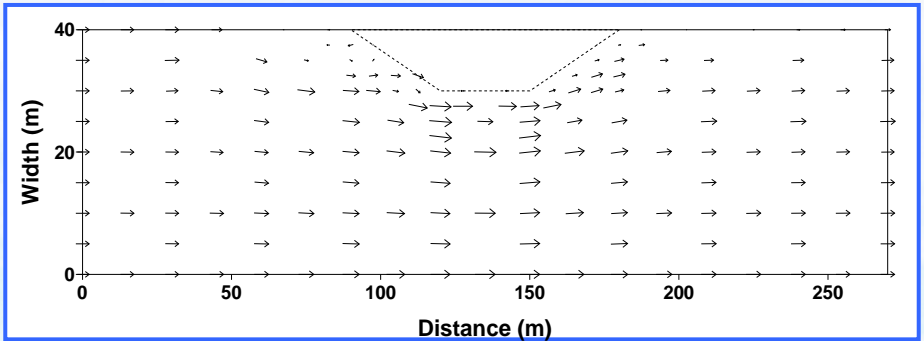
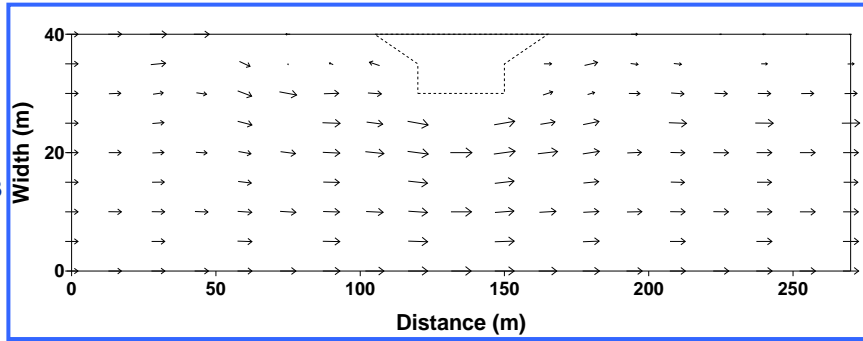


Velocity distribution

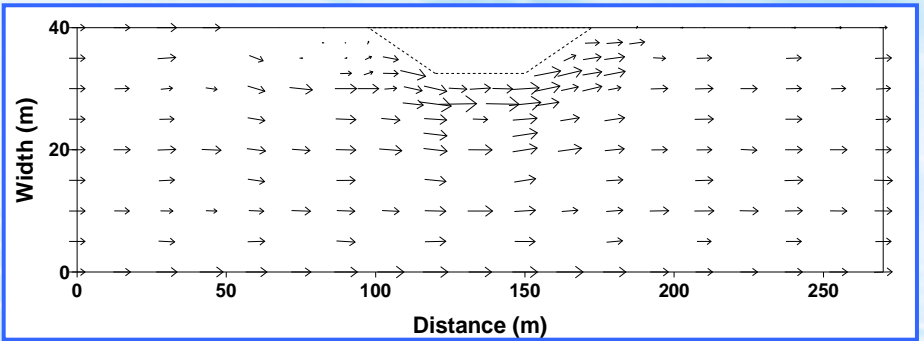
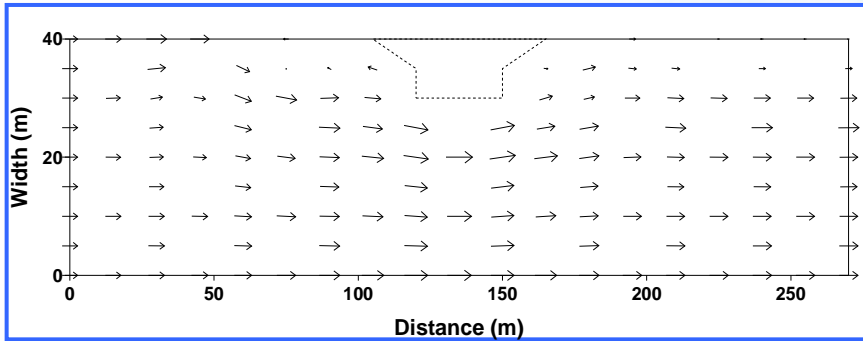


Depth contour

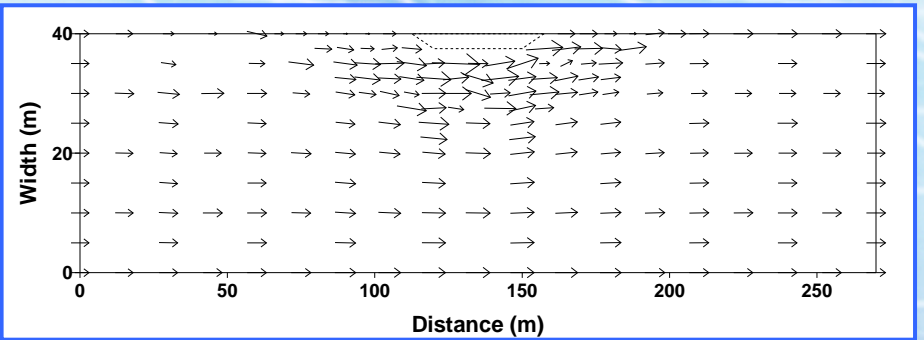
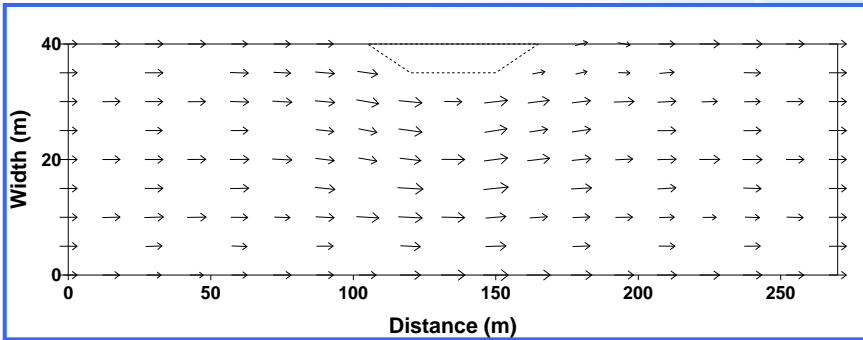
60
m/s³



80
m/s³



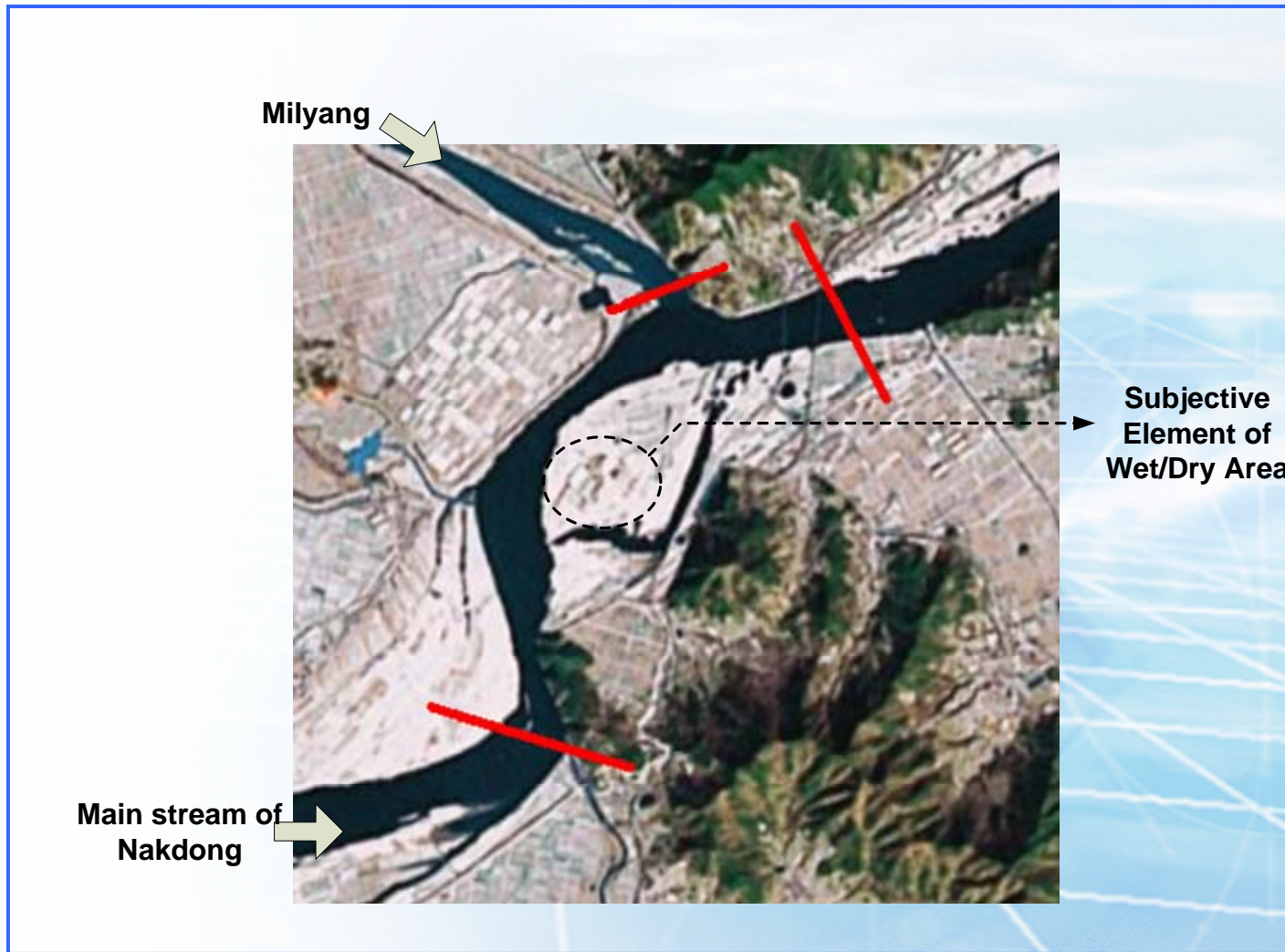
120
m/s³



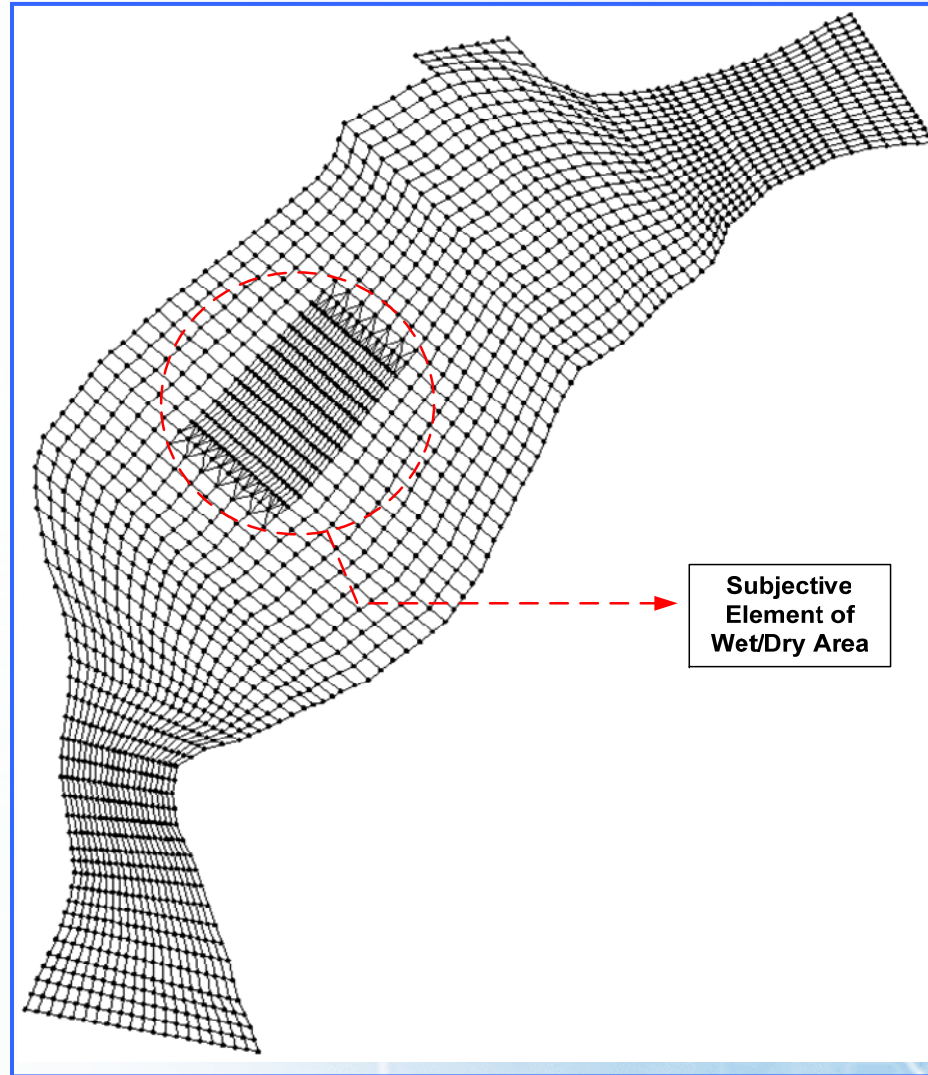
coarse element

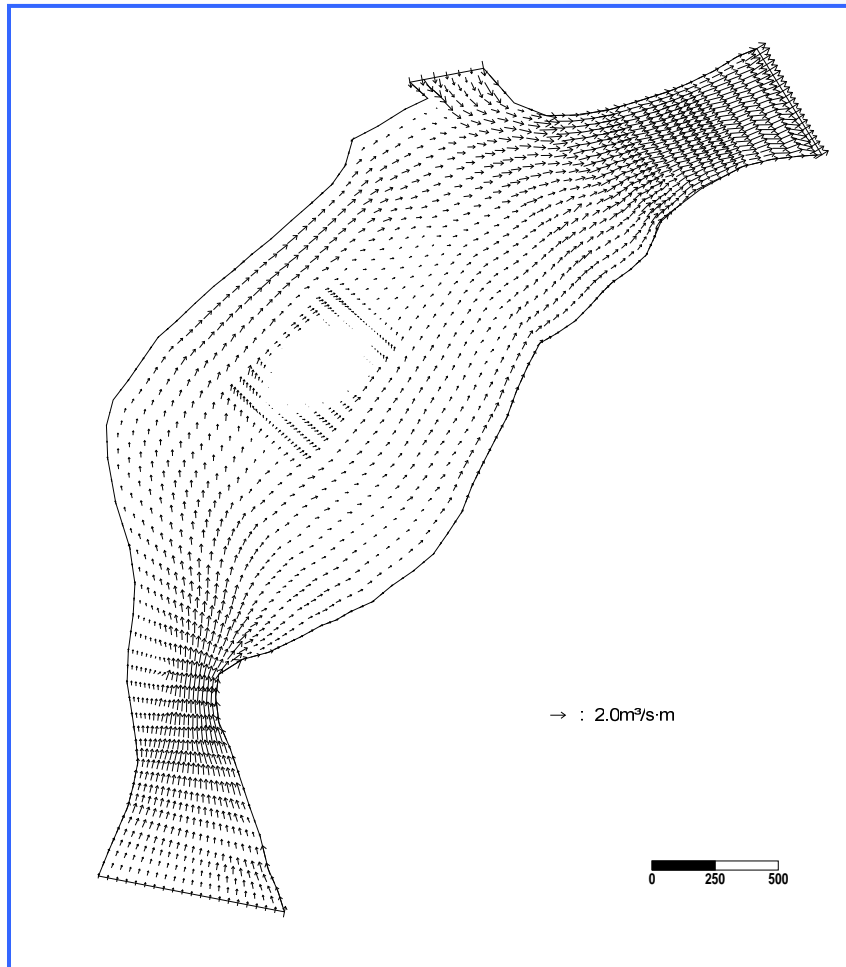
refined element

Application to Milyang-river

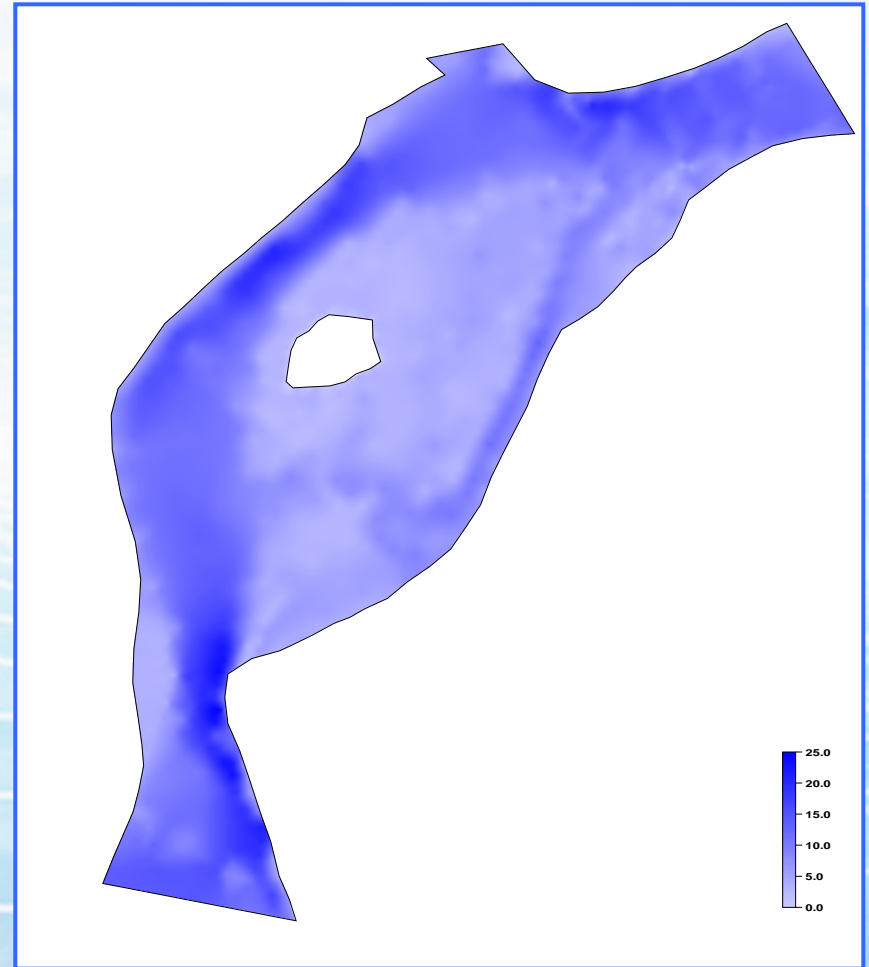


Milyang-river



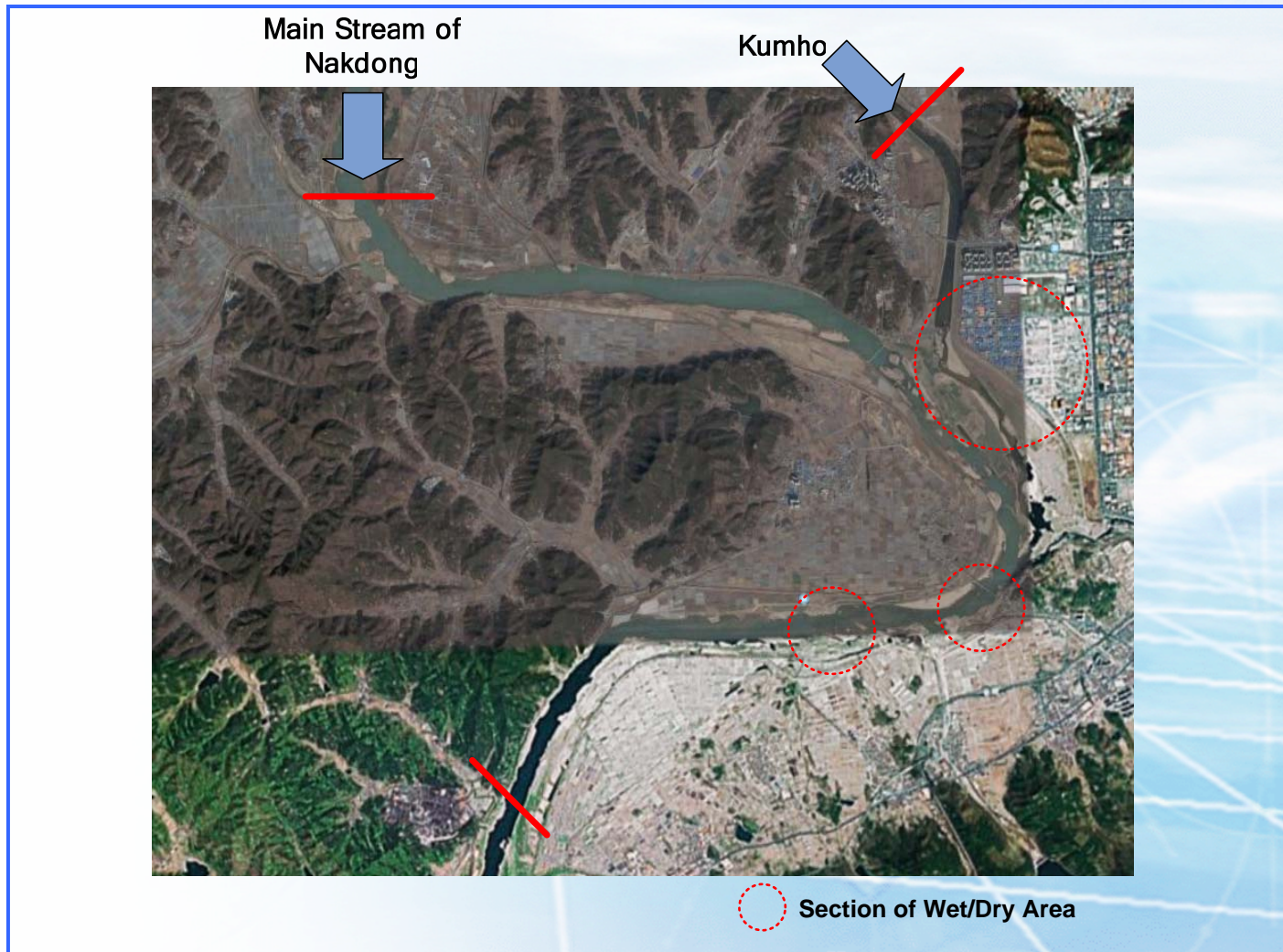


Velocity distribution

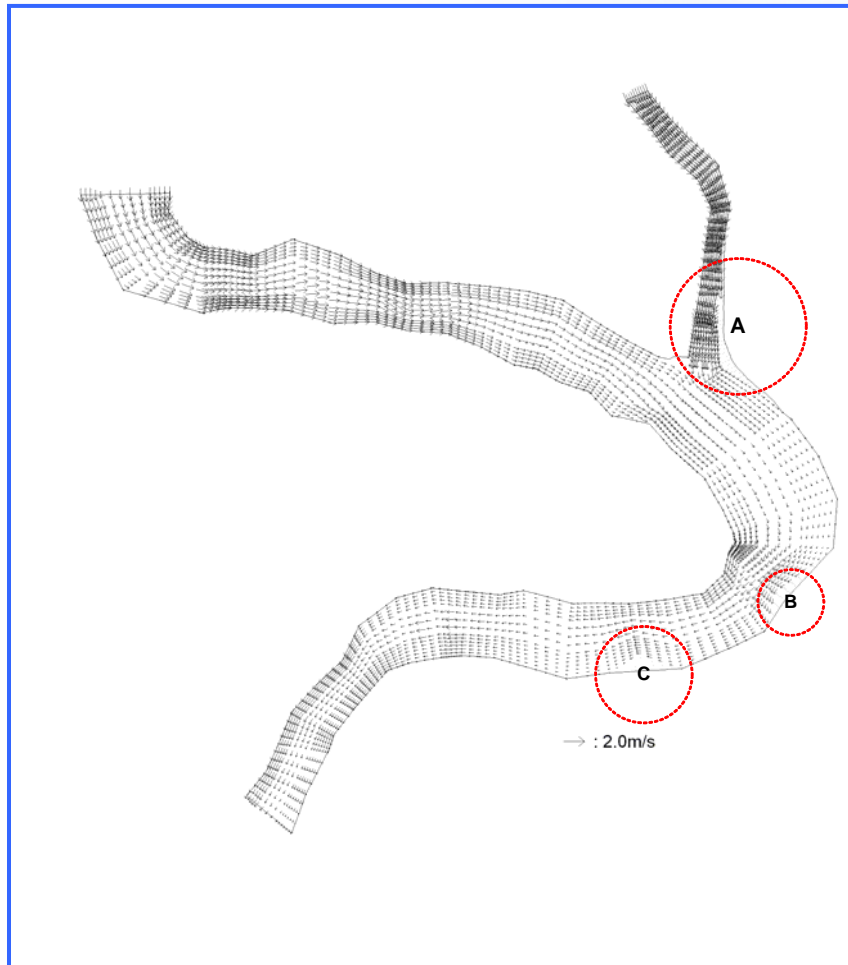


Water surface elevation

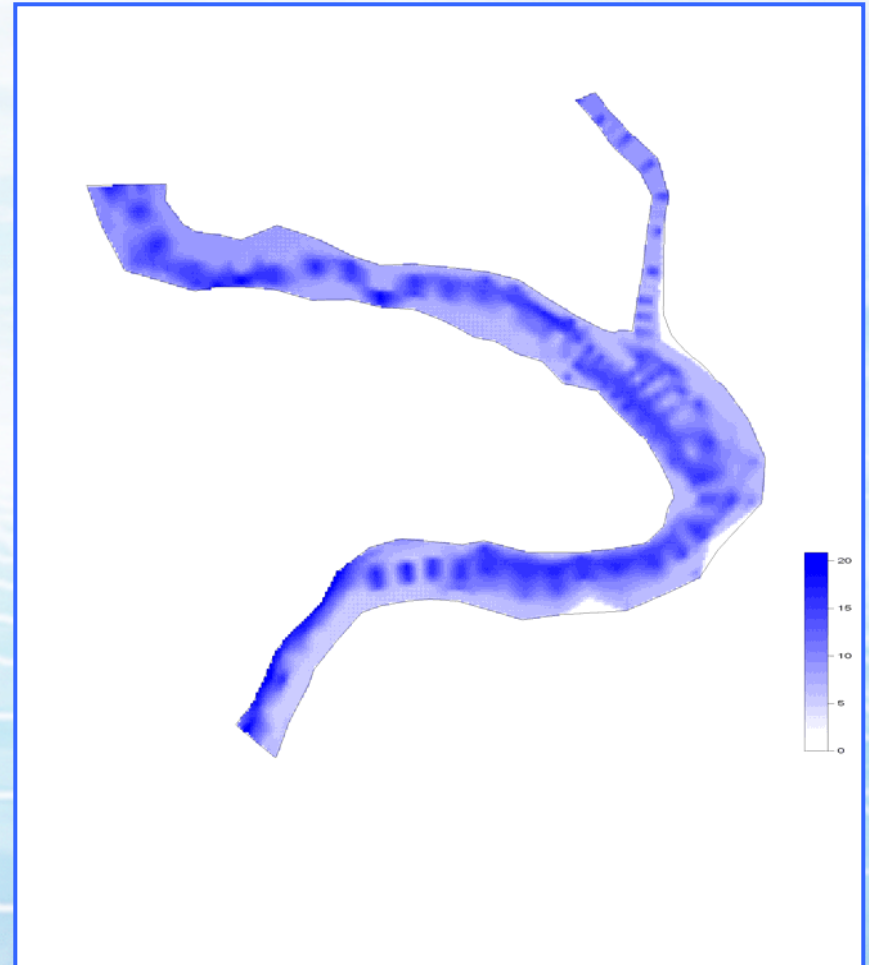
Application to Nakdong-river



Nakdong-river

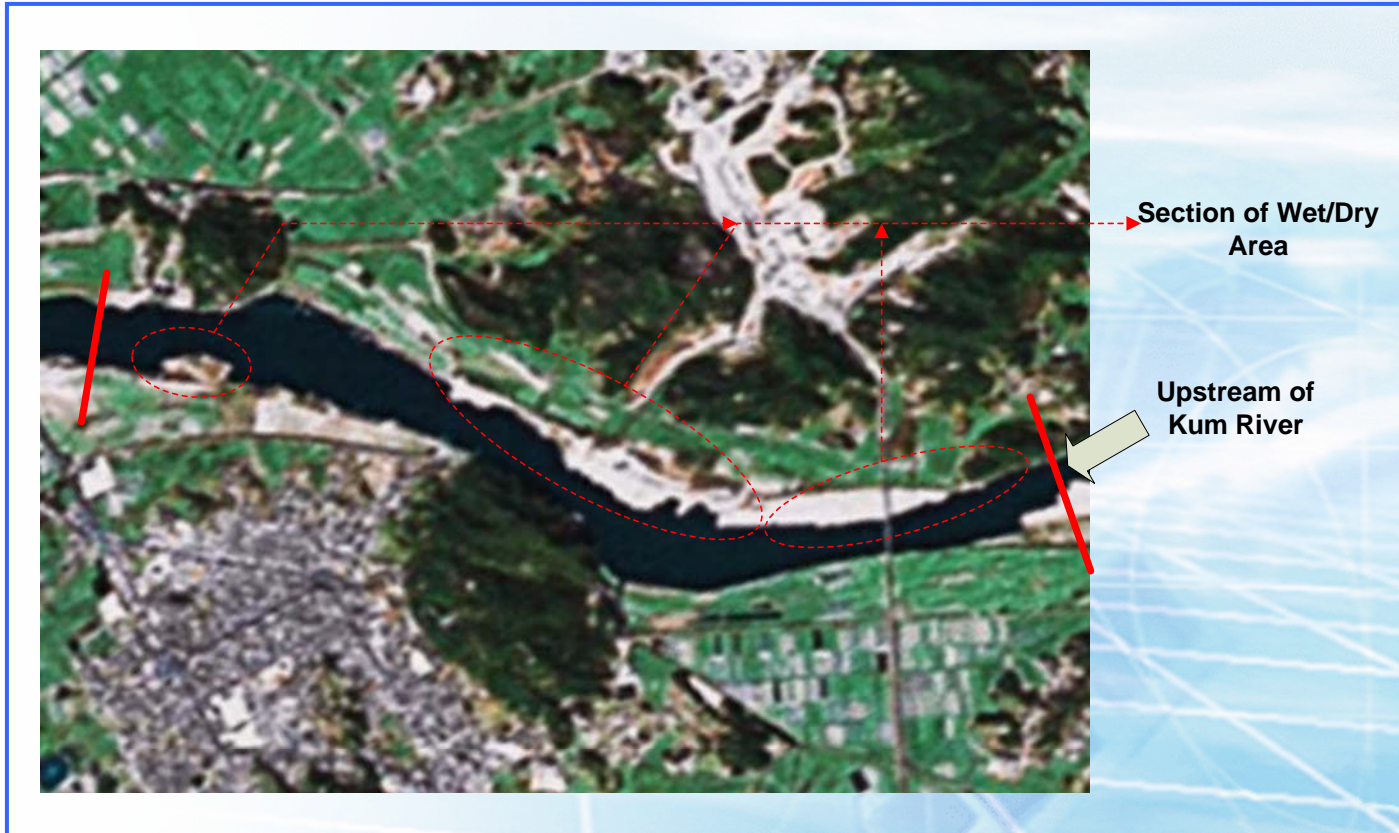


Velocity distribution

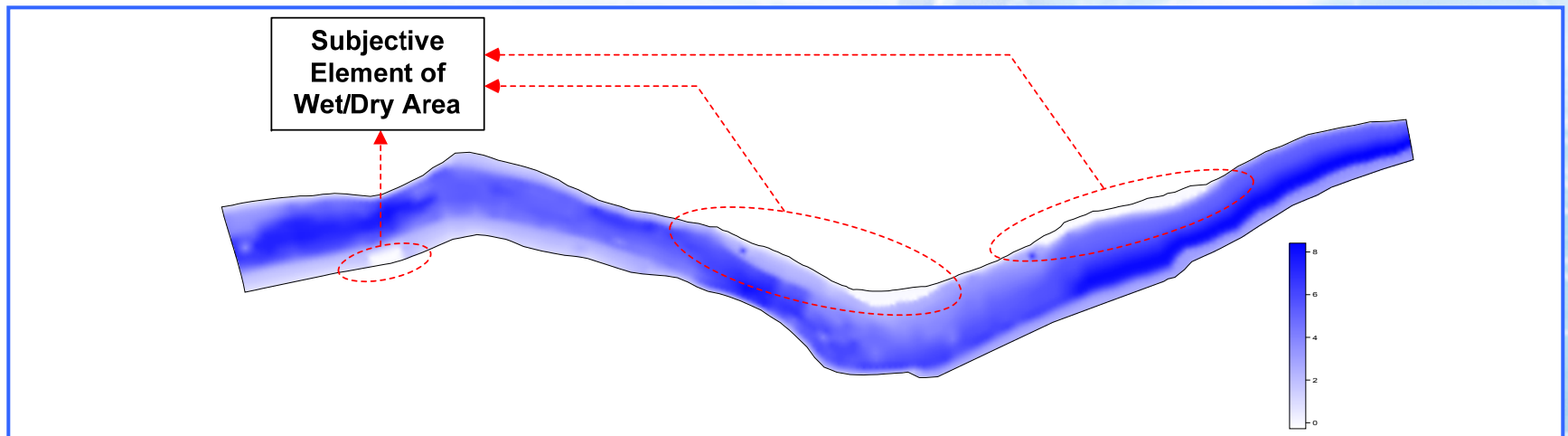
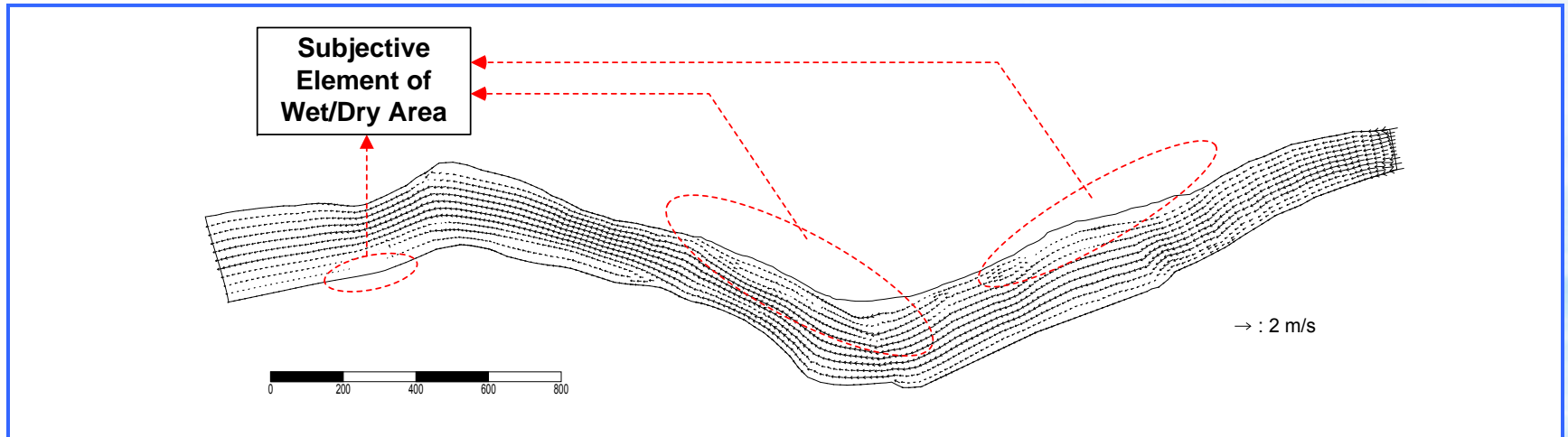


Water surface elevation

Application to Keum River



Kum-river



Velocity distribution and Water surface elevation

Conclusions(1)

- (1) The challenging problem facing two-dimensional model is the treatment of wet and dry areas. This situation is encountered in most practical river and coastal engineering problems.
- (2) To solve the dry/wet problems, deforming grid method, transition element method, and hybrid method are adopted.
- (3) The model is verified by applying to U-shaped laboratory channel. The simulation result agree with observed data for various flow conditions.

Conclusions(2)

- (4) Trapezoidal channel with partly dry side slopes, straight channel with various drying conditions, are examined for flow model validations.
- (5) RAM2 model shows reasonable flow distribution compared with existing model in dry area simulation in Milyang-river, Nakdong-river and Keum-river.
- (6) RAM2 model can be used as a hydrodynamic input for water quality and sediment transport model as well as be used for analyzing dam-break flow, levee-break flow and rapidly varied flow.

Thank You ..